The Candy Conundrum

Some years ago, a candy company advertised the large number of flavors that could be made by mixing their candies in your mouth. Here we can calculate how many there really are.

1. You have 5 red candies. How many different nonempty sets of candies could you put in your mouth?

2. You have 5 red and 4 green candies. How many different nonempty sets of candies could you put in your mouth?

3. You have 5 red, 4 green, and 3 yellow candies. How many different nonempty sets of candies could you put in your mouth?

We'll consider two sets of candies to be the *same flavor* if the ratio of candies of each color in one set is the same as the ratio in the other. For instance, 2 green and 1 yellow is the same flavor as 4 green and 2 yellow: 2/3 green, or a 2:1 green:yellow ratio. Similarly 3 red is the same flavor as 2 red: pure red!

4. You have 5 red candies. How many different flavors could you make?

5. You have 5 red and 4 green candies. How many different flavors could you make?

6. You have 5 red, 4 green, and 3 yellow candies. How many different flavors could you make?

There's a geometric interpretation of all of the above. For instance, with 5 red and 4 green candies, the possible combinations are ordered pairs. So (2,1) and (4,2) are the same flavor, for example.

7. Expressed geometrically, what does it mean for two different sets of candies to be the same flavor? Assume for now that there are only two colors.

8. Describe a geometric way of understanding how many different flavors there are. Compare it to the numeric approach.

9. What does symmetry tell you about the number of flavors with k red candies and k green candies?

Now let's try for some bigger patterns.

10. If you have 1 candy of each of *n* colors, how many different flavors are possible?

11. If you have *k* candies of 1 color, how many different flavors are possible? OK, sorry, that was too easy.

12. If you have 2 candies of each of *n* colors, how many different flavors are possible?

13. If you have *k* candies of each of 2 colors, how many different flavors are possible?

14. Generalize as much as you can!

15. How do the previous answers change if the candies are large, so there is an upper limit to how many can fit in your mouth at once?

16. What can you say about the relative probability of various flavors if you pick a random handful of size *n* out of a set of candies? Start by considering some easy cases, where *n* is small, and there aren't too many different flavors, and plenty of candies of each flavor (since *n* will limit you, it gets more complicated if you also have limits due to running out of candies).

17. What other questions can you think of about how to count combinations of candies?