Julia Robinson Mathematics Festival

Directed By Joshua Zucker

Cut!

Lines in the Plane

1. With 0 lines, the plane is in one piece. With 1 line, the plane is cut into two pieces. With 2 lines, what is the largest number of pieces the plane can be cut into? How about with 3 lines? And 4 lines? Can you generalize?

2. What is the smallest number of pieces the plane can be cut into with 2 lines? With 3 lines? And 4 lines? Can you generalize?

3. What are all possible numbers of pieces you can make with n lines?

4. What is the largest number of *bounded* pieces you can make with n lines?

5. What happens when your n lines are all formed by first drawing a regular n-gon and then extending the sides into lines? Is there a big difference between odd and even n?

Cutting the circle

6. What is the largest number of regions a circle can be cut into when n chords are drawn in it? (A chord is a segment whose endpoints are on the circle.)

7. What is the largest number of regions a circle can be cut into by first drawing n points on its edge, and then drawing all possible chords that connect those points?

8. In the previous problem, how many regions are produced when the n points are the vertices of a regular n-gon?

9. In all the previous circle problems, how many of the regions have at least one curved edge (or in other words, touch the outside of the circle)? How many have only straight segments for their edges? How many of the regions are triangles?

Cutting with circles

10. You can make Venn diagrams using circles that cut the plane into two regions with one circle, four regions with two circles, and eight regions with three circles. Is it possible to make sixteen regions with four circles?

11. What is the largest number of regions that can be made with n circles in the plane?

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Sponsored by the desJardins/Blachman fund

Cutting with zig-zags

12. A "zig-zag" is two rays pointing in opposite directions, with their endpoints joined by a line segment. How many regions can you divide the plane into by using two zig-zags?

13. Generalize. How many regions can you obtain with *n* zig-zags?

14. How many of those regions are bounded?

Cutting with planes

15. With 0 planes, space is in one piece. With 1 plane, space is cut into two pieces. With 2 planes, what is the largest number of pieces that space can be cut into? How about with 3 planes? And 4 planes? Can you generalize?

16. What is the smallest number of pieces that space can be cut into with 2 planes? With 3 planes? And 4 planes? Can you generalize?

17. What are all possible numbers of pieces you can make with *n* planes in space?

18. What is the largest number of *bounded* pieces you can make with *n* planes?

19. What happens when your *n* planes are all formed by first drawing a Platonic (regular) solid and extending all the face planes? (Recall that the Platonic solids are the tetrahedron with four triangular faces, the cube with six square faces, the octahedron with eight triangular faces, the dodecahedron with twelve pentagonal faces, and the icosahedron with twenty triangular faces.)