Fibonacci Flips

There are some methods that are pretty effective at distinguishing a sequence of coin flips generated by a human's imagination from one generated by flipping a fair coin. The idea for one or two of them is contained in the discussion below.

1. Write down a sequence of 100 H's and T's (or 1's and 0's, or any two symbols you like) to simulate flipping a coin 100 times. Then, as you do the problems below, compare the number of times each thing occurs with how often it should occur according to your probability calculations.

2. Out of all the places where the coin comes up heads, what fraction should be immediately followed by heads? By tails? Compare with your sequence.

3. Out of all possible lists of 100 flips, how many never have more than 1 consecutive flip the same?

4. Out of all possible lists of 100 flips, how many never have more than 2 consecutive flips the same?

5. Why is this activity called "Fibonacci Flips"?

6. For those of you who know matrix multiplication, how could you use it to help solve this problem?

7. Out of all possible lists of 100 flips, how many never have more than 3 consecutive flips the same?

8. Generalize: how would you find the probability of never having 6 consecutive flips the same? (Does your sequence that you invented in problem 1 ever have 6 consecutive flips the same?)

9. On average, how many flips does it take before you first get 1 in a row the same? 2 in a row? 3 in a row? Can you generalize? Visit the "Infinity Finite" table if this question interests you.

Julia Robinson Mathematics Festival

Directed by Joshua Zucker
Castilleja School

H		T	
HH		TH	
HT		TT	
HHH		THH	
HHT		THT	
HTH		TTH	
HTT		TTT	
HHHH	HTHH	THHH	TTHH
HHHT	HTHT	THHT	TTHT
HHTH	HTTH	THTH	TTTH
HHTT	HTTT	THTT	TTTT
HHHHH	HTHHH	THHHH	TTHHH
HHHHT	HTHHT	THHHT	TTHHT
HHHTH	HTHTH	THHTH	TTHTH
HHHTT	HTHTT	THHTT	TTHTT
HHTHH	HTTHH	THTHH	TTTHH
HHTHT	HTTHT	THTHT	TTTHT
HHTTH	HTTTH	THTTH	TTTTH
HHTTT	HTTTT	THTTT	TTTTT
HHHHHH	HTHHHH	THHHHH	TTHHHH
HHHHHT	HTHHHT	THHHHT	TTHHHT
HHHHTH	HTHHTH	THHHTH	TTHHTH
HHHHTT	HTHHTT	THHHTT	TTHHTT
HHHTHH	HTHTHH	THHTHH	TTHTHH
HHHTHT	HTHTHT	THHTHT	TTHTHT
HHHTTH	HTHTTH	THHTTH	TTHTTH
HHHTTT	HTHTTT	THHTTT	TTHTTT
HHTHHH	HTTHHH	THTHHH	TTTHHH
HHTHHT	HTTHHT	THTHHT	TTTHHT
HHTHTH	HTTHTH	THTHTH	TTTHTH
HHTHTT	HTTHTT	THTHTT	TTTHTT
HHTTHH	HTTTHH	THTTHH	TTTTHH
HHTTHT	HTTTHT	THTTHT	TTTTHT
HHTTTH	HTTTTH	THTTTH	TTTTTH
HHTTTT	HTTTTT	THTTTT	TTTTTT

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