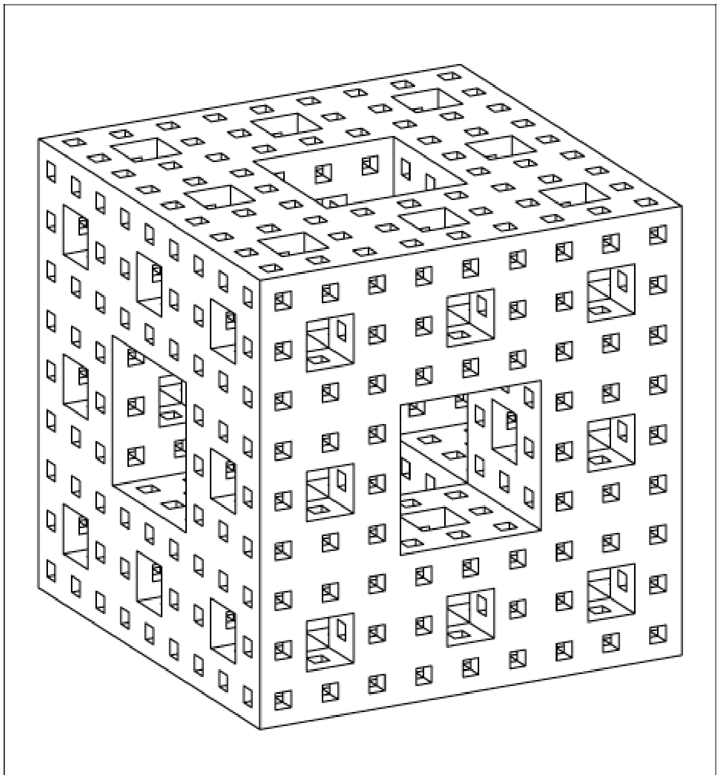
Menger Sponge

Take a cube, divide it into 27 (3 x 3 x 3) smaller cubes of the same size; now remove the cube in the center of each face plus the cube at the center of the whole. You are left with a structure consisting of the eight small corner cubes plus twelve small edge cubes holding them together. Now, imagine repeating this process on each of these remaining 20 cubes. Repeat again. And again, ad infinitum ...



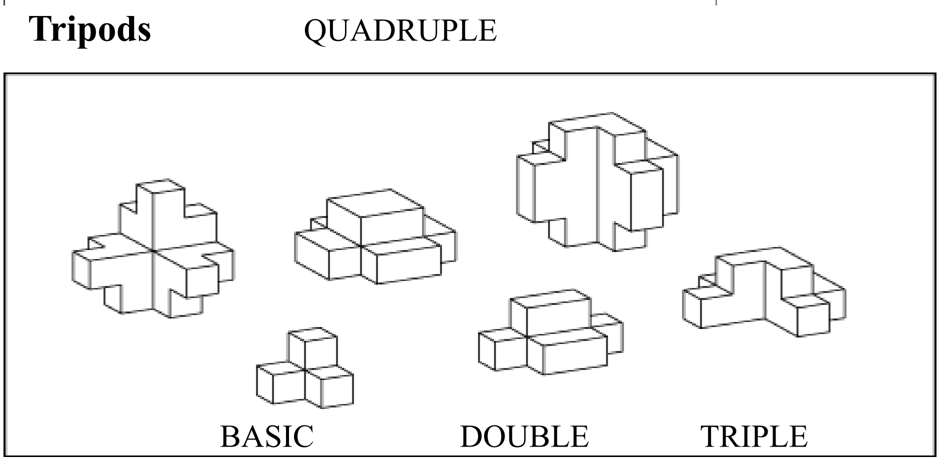
1. Build a Menger sponge out of business card cubes. One cube is level zero. Can you build a level 1 sponge?

2. Can you build a level 1 sponge out of 4 single tripods and 4 additional cubes?

3. Can you build a level 2 sponge? Does a big collection of level 1 sponges help?

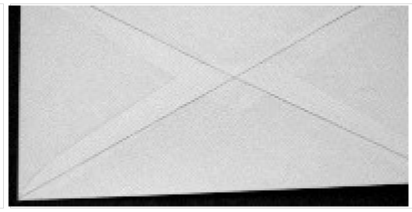
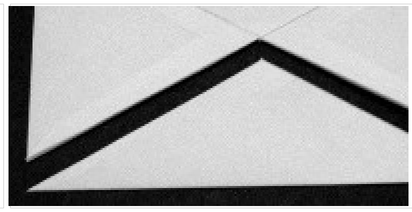
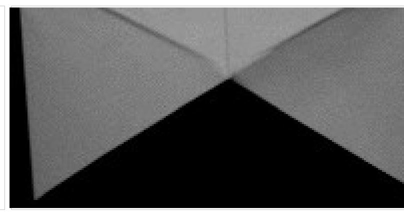
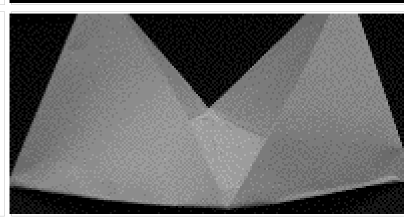
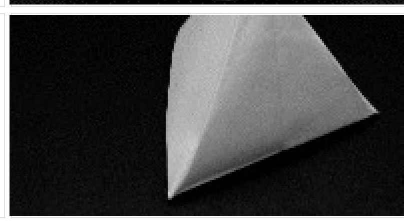
4. Bonus: What do you get if you slice through the center of the sponge? If your slice is through the midpoints of the edges?

Tripods



Sierpinski Tetrahedron

First we will discuss how to create a unit tetrahedron.

First fold	
First cut	
Fold again	
Open and fold again	
Open and fold again	

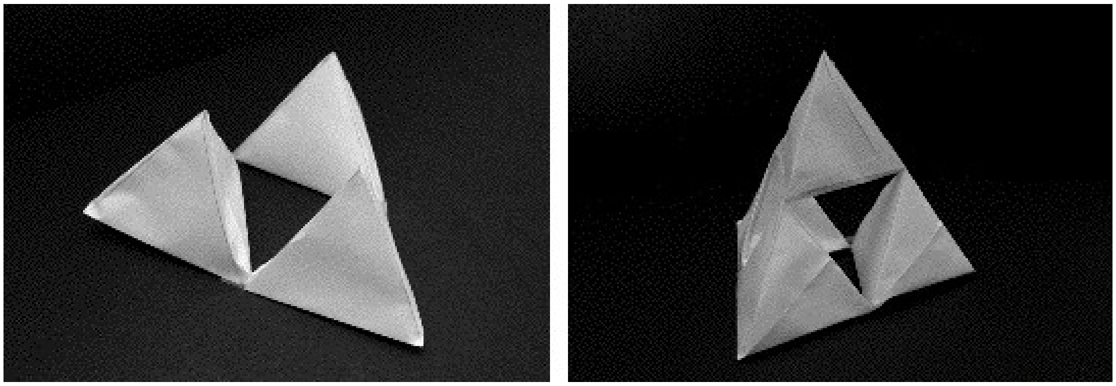
Having made individual tetrahedrons, now tape four together to make a larger shape.

To do this place and tape three small tetrahedrons together to form a base, then place and tape the fourth on top to the other three (like a pile of cannon balls). This will look like a pyramid with a hole in the middle.

Then take four of these shapes and make a larger shape in the same manner. At this stage we have used sixteen tetrahedrons.

Finally, put four of these shapes together to make the completed tetrahedron using sixty-four envelopes.

To go one more stage will require 256 envelopes in total and a lot of tape, time, patience, cleverness and cooperation to make it all fit.



The base and the completed first stage tetrahedron

Julia Robinson Mathematics Festival

Directed By Joshua Zucker

FRACTAL DIMENSION

Fractals are self similar objects. The Menger sponge contains cubes, all similar, at different scales. The Sierpinski Tetrahedron contains similar tetrahedrons at different scales.

Fractal Dimension Definition: $n = m^d$

n = number of self similar pieces

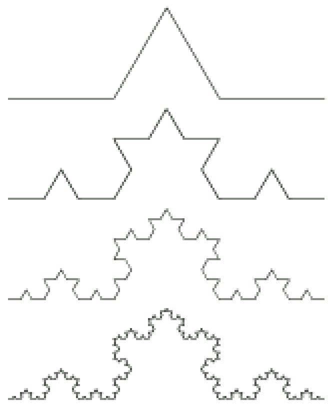
m = magnification factor

d = dimension

1. Show that for a line segment, a square and a single solid cube, that this definition gives the ordinary dimension.
2. How do you solve the equation above for d?
3. Suppose we construct a Menger sponge by starting with a cube and taking away material as in the original description above. If we continue this process many times, what will happen to the volume and surface area? What is the dimension of the Menger Sponge?
4. Suppose we construct a Sierpinski tetrahedron by starting with a tetrahedron and taking away smaller and smaller tetrahedrons. If we continue this process many times, what will happen to the volume and surface area? What is the dimension of the Sierpinski tetrahedron?

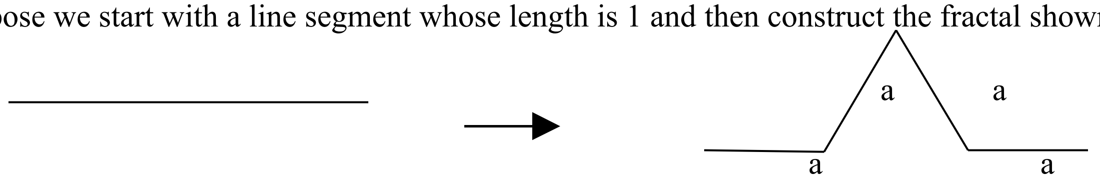
Credits for these Fractal problems: Bruce Amundson
http://www.theiff.org/oexhibits/menger02.html
http://classes.yale.edu/Fractals/Labs/SierpTetraLab/SierpTetraLab.html
Fractal, A Tool Kit of Dynamics Activities, Jonathan Choate, Robert L. Devaney, Alice Foster

KOCH CURVE



Shown are stages 0 to 4 of the Koch curve. Each is obtained from the previous by adding segments 1/3 the length of the previous segment. Suppose this process is repeated indefinitely.

1. What is the length of the curve? Why?
2. What is the area under the curve? How did you compute it ?
3. What is the fractal dimension?
4. Suppose instead of 1/3, the segments are 2/7 the length of the previous segment. What is the fractal dimension now?
5. Suppose the segments are 2/5 the length of the previous segment. Fractal dimension?
6. Describe a way to construct a fractal with the fractal dimension of any given number between 0 and 1.
7. Suppose we start with a line segment whose length is 1 and then construct the fractal shown here:



Each of the four pieces has a length of a, where a is a number between ¼ and ½ . What is the fractal dimension of the object?

When a = ¼, what is the object? What is the fractal dimension?
When a = ½ , what is the object? What is the fractal dimension?

If you made a movie and each frame of the film was a picture of this object as a increases, what would you see?

8. Given any number between 1 and 2, describe a way to construct a fractal whose dimension is exactly that number?