The Gossip Problem

Everyone knows something that nobody else knows. But everybody else wants to know! That's why we need gossip.

To make that a bit more mathematical: person #1, 2, 3, ..., \( n \) each know a unique piece of information. When two people talk, they “gossip”, exchanging all the pieces of information they have acquired. How many conversations (with two people at a time) are necessary in order that everyone will know all the information?

1. With 1 person it's too easy: no talking is necessary, the 1 person already knows everything.

2. With 2 people it's still pretty easy: one conversation and they exchange their information.

3. How about with 3 people? One conversation clearly isn't enough. Can they do it with 2, or do they need all three?

4. With four people things start to get interesting. There are 6 possible conversations: are they all necessary? What strategies can you use to avoid unnecessary conversations?

5. What happens with five people?

6. Can you find a strategy with \( n \) people that, instead of the \( (n^2 - n)/2 \) possible conversations, takes a lot less - like only \( 2n \)?

7. Can you, at least for large \( n \), do it with \( 2n - 1 \)?

8. How about \( 2n - 3 \)?

9. Can you do \( 2n - 4 \)?

10. Can you beat \( 2n - 4 \)?

11. What if your goal, rather than minimizing the total number of conversations, is to minimize the maximum number of conversations that any one person has to participate in? Then how do you do it, how many total conversations does it take, and what is the maximum for one person?

12. What if everyone has a cell phone and can talk to one other person at a time? How do you minimize the total amount of time it takes to finish? (You don't want people waiting around for other people to make a bunch of calls.)