Julia Robinson Mathematics Festival

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The Gossip Problem

Everyone knows something that nobody else knows. But everybody else wants to know! That's why we need gossip.
To make that a bit more mathematical: person #1, 2, 3, ..., n each know a unique piece of information. When two people talk, they "gossip", exchanging all the pieces of information they have acquired. How many conversations (with two people at a time) are necessary in order that everyone will know all the information?

1. With 1 person it's too easy: no talking is necessary, the 1 person already knows everything.

2. With 2 people it's still pretty easy: one conversation and they exchange their information.

3. How about with 3 people? One conversation clearly isn't enough. Can they do it with 2, or do they need all three?

4. With four people things start to get interesting. There are 6 possible conversations: are they all necessary? What strategies can you use to avoid unnecessary conversations?

5. What happens with five people?

6. Can you find a strategy with n people that, instead of the $(n^2 - n)/2$ possible conversations, takes a lot less - like only $2n$?

7. Can you, at least for large n, do it with $2n - 1$?

8. How about $2n - 3$?

9. Can you do $2n - 4$?

10. Can you beat $2n - 4$?

11. What if your goal, rather than minimizing the total number of conversations, is to minimize the *maximum* number of conversations that any one person has to participate in? Then how do you do it, how many *total* conversations does it take, and what is the maximum for one person?

12. What if everyone has a cell phone and can talk to one other person at a time? How do you minimize the total amount of time it takes to finish? (You don't want people waiting around for other people to make a bunch of calls.)

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Sponsored by the desJardins/Blachman fund