Partner Puzzles

The Square Dance

1. Each number from 1 through 16 seeks a partner for a square dance. Because it's a square dance, of course, each number when added to its partner must equal a perfect square. Is this possible? If not, why not? If so, is there more than one way?

2. What if, instead of the numbers 1 through 16, you have 1 through 2? Well, that obviously won't work (why not?). How about 1 through 4?

3. When is it possible to assign numbers 1 through n into pairs for a square dance? When is it impossible? Justify your answers.

More Dancing

4. The six numbers 1 through 6 are in a dance. They stand in a three by two rectangle such that the seven sums of each vertically or horizontally adjacent pair of numbers are all different. Can you arrange the numbers to make this happen?

5. How many different ways are there to arrange these numbers in their rectangle? (We count rotated or reflected copies of a given arrangement to be the same.)

6. Now the six numbers dance such that, instead of having seven different sums, there are as few different sums as possible. What is the smallest possible number of different sums, and how do you know there can't be fewer?

7. How many ways are there to arrange the numbers 1 through 6 in order to reach this fewest possible number of different sums?

8. Now there are nine numbers, 1 through 9, at the dance. Can they be arranged in a square so that the twelve sums of vertically or horizontally adjacent pairs are all different?

9. Can the numbers 1 through 12 be arranged in a three by four rectangle so that all the sums of horizontally or vertically adjacent pairs are different?

10. Generalize. Can the numbers 1 through mn be arranged in an m by n rectangle so that all the sums of horizontally or vertically adjacent pairs are different?

Prime Arrangements

11. Arrange the numbers 1 through 16 into a 4 by 4 square so that the sum of each vertically or horizontally adjacent pair of numbers is a prime number (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, or 31).

12. Can you do it without using 31 as one of your sums?

13. How many sums are there in the grid, that you would need to check in order to be sure they are all prime?

14. Now repeat the process with a 5x5 square.

15. What if all the sums must be at least 11 and at most 41?