Pirate Puzzles

The pirates in these problems are all very good logicians, of course!

1. Three pirates are all wearing hats. Some of them may have secretly had the skull and crossbones removed from their hats. The pirate captain says "Two of you are going to walk the plank – the first one of you scalawags to correctly determine whether your hat has the skull and crossbones gets to be my first mate. Now, raise your hand if you see at least one skull and crossbones!"

    All three pirates raise their hands, but no pirate immediately knows whether their hat has the skull and crossbones. The smartest pirate then knows and becomes first mate. What did they know, and how did they know it?

2. The captain walks in on a group of three lazy but very intelligent pirates. "So, mateys, you will all walk the plank!" The men plead for an extra chance, and the captain replies, "Aye, in a few minutes I'll put a hat on each of your heads, randomly picking either white or black. Ye'll be able to see the color of the other two hats but not your own. Then without any blabbing, ye'll each write on a piece of paper, either 'my hat is black' or 'my hat is white' or 'I do not want to guess'. If at least one of you guesses, and if everyone who guesses is correct, then I'll spare the lot o' ye." The pirates spend their few minutes frantically thinking about the best strategy. Who should guess? How should they make their guess? Finally they find a strategy that gives them a 75% chance of survival. How do they do it?

3. Ten pirates stand in a line, one facing the next, so the first pirate can see the hats of all the other nine, and the second pirate can see eight, and so on. Again, the malicious captain threatens to throw them all overboard, and is persuaded to put one of two hats on each pirate at random, and to give them a few moments to discuss strategy. Now the pirates survive only if all ten guess the color of their hat correctly. How should they choose their guesses?

4. Ten pirates are fighting over a treasure of 100 gold coins. Each pirate in turn gets to propose a plan for dividing the treasure, whereupon all the remaining pirates vote. If half or more of the pirates vote in favor of the plan, then they carry it out. Otherwise, the first pirate is thrown overboard, and then the second pirate proposes a plan. The pirates are, of course, superintelligent, maximally greedy for themselves, wanting to live and get the most gold they can. All else being equal they'd rather throw someone overboard, but not if it costs them any gold. The coins are unsplittable. The pirates aren't able to trust each other enough to make bargains outside of the voting process. What should the first pirate propose, and what should happen?

5. How does the answer to the previous problem change when there are more than 10 pirates? More than 100? More than 200? What's the pattern for very large numbers of pirates of what the first pirate should propose?

6. How does the answer change if the pirate only avoids being thrown overboard when the proposed division of the loot gets more than half of the votes?