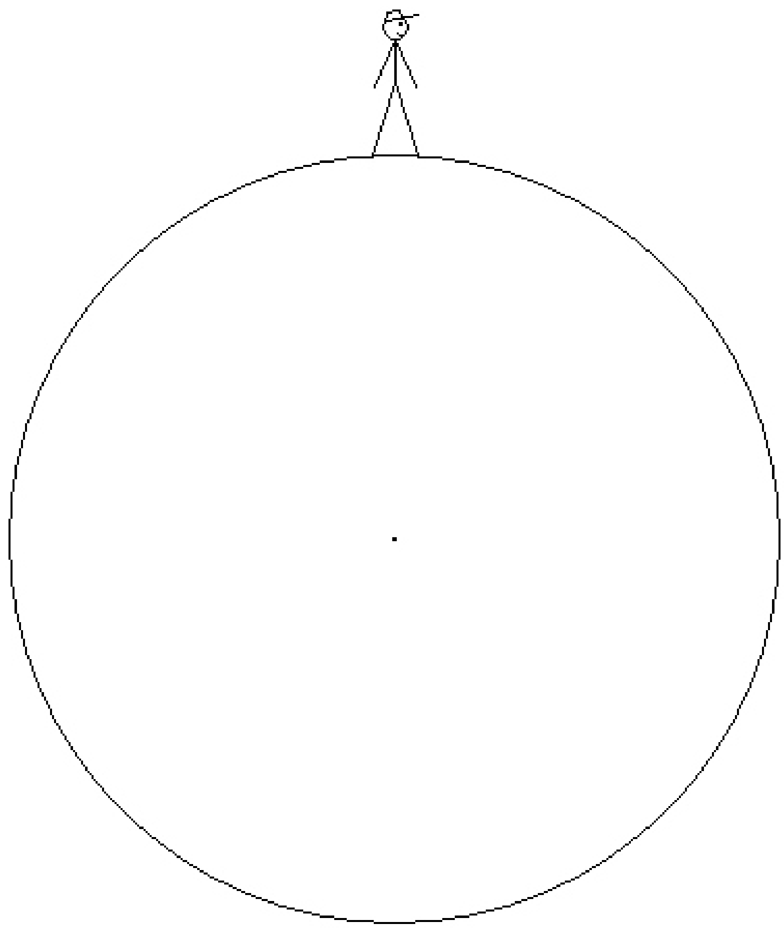
# Staring out to Sea...



Miles is at the beach, standing on the shoreline and looking out at the horizon. All of a sudden he begins to wonder just how far he can see...

1. Above is a two-dimensional, not-to-scale diagram of Miles standing on the earth. Mark the area on the earth's surface that Miles can see. (Miles can see a point if there is a line from that point to his eye, which is not blocked by the earth. Use a straightedge.)

2. Label the horizon (the farthest point that Miles can see) "B." Label Miles's eye "A" and the center of the earth "C." Draw in triangle ABC. What is angle ABC? Why?

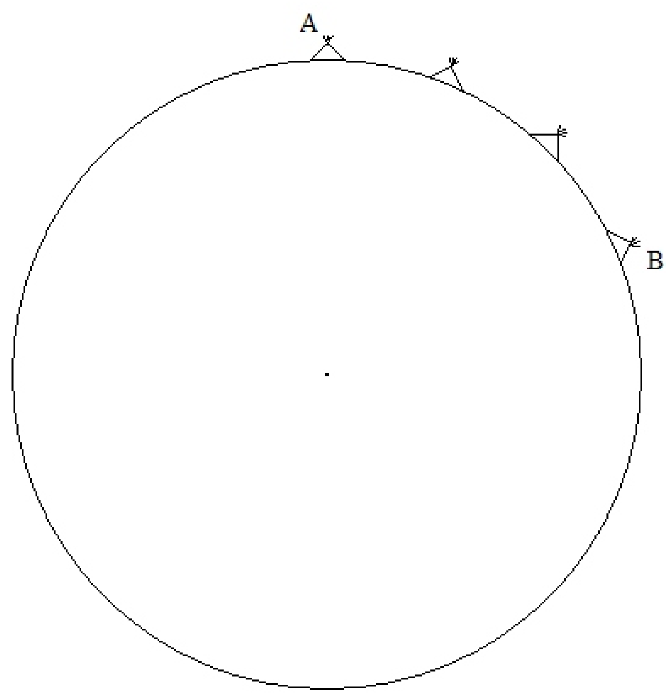
3. Assume that Miles is 1 m. tall and the earth's radius is 6,000 km. (6,000,000 m). How far can he see? Can you write a formula for the distance to the horizon in terms of the earth's radius r and a person's height h?

4. Check your formula by testing it in some special cases: $h = 0$, $h = r$, $r = 0$, $h = 10r$. Do your answers make sense?

5. The radius of the earth in Mountain View (that's right, radius differs slightly at different latitudes–the earth isn't a perfect sphere) is actually about 6,370 km. Use this value to calculate the distance to the horizon for your own height. You will need to convert your height to meters. One foot = 0.3048 m. One inch = 0.0254 m. If you want your final answer in miles, use the ratio 1 km. = .6214 miles.

6. What are some factors that might make your calculation in problem 4 inaccurate? How do you think an experimental answer would compare to your theoretical answer?

Miles's favorite scene in the *Lord of the Rings* movies is when two distant kingdoms communicate by a chain of signal fires. First a hobbit in Kingdom A lights a bonfire on a mountaintop; the fire is visible on another faraway mountaintop, where another messenger lights another fire; the process continues until the message has spread far beyond the horizon of Kingdom A.




7. Assume again that the earth is a perfect sphere of radius 6,000 km. Say that each mountain is 1 km. high. What is the maximum distance between two successive mountains so that the signal fire is still visible?

8. How many fires would it take to send a message around the circumference of the earth, from Kingdom A back to Kingdom A?

9. How high would the mountains have to be if only three fires were required for problem 8? What if only four fires were required? What about n fires for any integer n?

The next problems deal with a measurement that is a little bit trickier than the straight-line distance from your eyes to the horizon. If you haven't learned trigonometry yet, skip to problem 12.

10. Write a formula for the distance along the earth's circumference from a Miles's feet to the horizon. You won't need any new information besides the earth's radius r and Miles's height h, but now you'll need to think about the angle ACB and the formula for circular arc length.

11. How does the measurement in problem 8 compare to the original eyes-to-horizon measurement when h is very small compared to r? Why? How does the feet-to-horizon measurement behave when h is very large compared to r? Why?

Finally, some horizon problems for other planets.

12. Suppose that you are standing with your feet at the point (0, 1) on an elliptical planet. Your height is h and the equation of the planet's surface is

$$\frac{x^2}{4} + y^2 = 1$$

What is the distance from your eye to the horizon?

13. Suppose that you are standing with your feet at (0, 0) on a parabolic planet. Your height is h and the equation of the planet's surface is $y = -x^2$

What is the distance from your eye to the horizon?

14. Invent and solve more problems of this style.