Julia Robinson Mathematics Festival

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To Twos, Too! Two Twos? More?

It's possible to write some numbers, like 5, as the Sum Of at least two Consecutive Positive Integers: 2+3. We'll call this a SOCPI representation. Some numbers have more than one SOCPI representation, like 15 = 1+2+3+4+5 = 4+5+6 = 7+8 = … is that all of them? (By the way, in this problem "number" means "positive integer".)

1. Find a number that has exactly two SOCPI, or explain why one doesn't exist.

2. Is there a number with exactly three SOCPI?

3. Is there a number with exactly four SOCPI?

4. For what values of k does there exist a number with exactly k SOCPI representations?

5. Neither 1 nor 2 have any SOCPI representations; they are just too small. 3 = 1+2. What numbers larger than 3 have no SOCPI representations?

6. Construct a formula for, or a method of determining, the number of SOCPI representations of any given positive integer n.

Some numbers, like 5, can be written as the Sum of Distinct Powers of Two: $2^2 + 2^0$. Distinct here means that you can use each power of 2 at most once. Numbers like 5 are OK, too: 2^2. You don't have to use more than one power of 2. We'll call these ways of writing a number SDP2 representations.

7. Find a number that has exactly two SDP2 representations, or explain why one doesn't exist.

8. Is there a positive integer that has no SDP2 representations?

9. For what values of k does there exist a number with exactly k SDP2 representations?

10. Construct a formula for, or a method of determining, the number of SDP2 representations of any given positive integer n.

Every number, like 5, can be represented as the Sum of (not necessarily distinct) Powers of 2: $5 = 2^2 + 2^0 = 2^1 + 2^1 + 2^0 = 2^1 + 2^0 + 2^0 + 2^0 = 2^0 + 2^0 + 2^0 + 2^0 + 2^0$. So 5 has four SP2 representations.

11. Find a number that has exactly two SP2 representations, or explain why one doesn't exist.

12. For what values of *k* does there exist a number with exactly *k* SP2 representations?

13. Construct a formula, or a method of determining, the number of SP2 representations of any given positive integer *n*.

Every number, like 5, can be represented as the Sum of at most 2 copies of (not necessarily distinct) Powers of 2: $5 = 2^2 + 2^0 = 2^1 + 2^1 + 2^0$. (The other examples from the previous section are no longer allowed, because there are more than 2 copies of one of the powers of 2). We'll call these S2P2 representations.

14. Find a number that has exactly one S2P2 representation, or explain why such a number does not exist.

15. For what values of *k* does there exist a number with exactly *k* S2P2 representations?

16. Construct a formula, or a method of determining, the number of S2P2 representations of any given positive integer *n*.

Now let's think about some generalizations.

17. What is different if we allow negative integers (and 0) in the first group of problems?

18. What happens if we consider powers of 3 throughout instead of powers of 2?

19. How about powers of 10?

20. What other related problems can you think of?

Thanks to The Math Less Traveled, http://mathlesstraveled.com, for the ideas for several of these problems.