

# Un-Holey Cubes (Conway Cube)

This type of cube dissection is believed to have first been studied by John Conway, who is also the inventor of the "stick" version. Whether he or someone else invented the  $3 \times 3 \times 3$  version (with 3 "holes" or  $1 \times 1 \times 1$  cubes) is not known, but it is reputed to have first been published in 1970 in a book called *Cubics*, by Jan Slothouber and William Graatsma. So the  $3 \times 3 \times 3$  version is sometimes called the Slothouber-Graatsma Cube. I've chosen to call this type of puzzle Un-Holey Cubes, because it needs a short name, and because the solution depends on figuring out where the "holes" (sometimes just the small, most odd-shaped pieces) go.

1. Try to fit all the pieces into the cube.
2. Look at the pieces: what are their dimensions?
3. Look at the size of the box: how big is it?
4. How can pieces (or part of the pieces) fit into **one layer** of the cube?  
Is there an extra hole, that can't be filled by normal pieces?
5. How many layers are there, from top to bottom?  
How many layers from front to back?  
How many from right to left?
6. If **every layer** must have a  $1 \times 1$  hole, and all layers must have a hole, and there are only 3  $1 \times 1 \times 1$  small cubes, where **must** the holes be?
7. You should now be able to solve the  $3 \times 3 \times 3$  puzzle.

## VARIATIONS

1. Try the  $5 \times 5 \times 5$  version, with 5  $1 \times 1 \times 1$  small cubes.
2. Try the  $7 \times 7 \times 7$  version with  $1 \times 1 \times 1$  small cubes.
3. With the  $3 \times 3 \times 3$  cubes, there is one without the  $1 \times 1 \times 1$  cubes, and one with the "holes" attached to the  $2 \times 2 \times 1$  pieces, and one where all the pieces are rhombic, at an angle. Which of the 4 versions is easiest? Which hardest? The one with no small cubes has fewest pieces, does that make it easier than the others?
4. Stick cubes (Conway Cubes): find the version that is a  $5 \times 5 \times 5$  box, with three  $1 \times 1 \times 3$  pieces. Try to solve it, with the knowledge you now have. Now the  $7 \times 7 \times 7$  Conway Cube, with 7 holes, should be easy!
5. There is also a large plastic (half) box, with clear plastic pieces. Make sure the box is  $7 \times 7 \times 7$  and that you have 14 of the  $4 \times 4 \times 1$  squares. The other pieces should be enough to exactly fill the  $7 \times 7 \times 7$  cube. This version was invented by Bill Gosper.