

Who Took the Candy?

In all the games described below there are two players, Alice and Bob, and Alice always plays first. The problem is to decide which one of the two players has a winning strategy (and, of course, to describe this strategy).

We recommend that you start with the first game, play it several times and then answer all the questions. Then go to the next game and do the same. The games become more challenging (and more interesting!). To play the games, you may use the provided plastic chips instead of candy.

1. There are N candies on a table. During each turn, a player must take exactly one candy. The player who takes the last candy wins.

Questions:

- (i) Who has a winning strategy if $N = 2$? If $N = 3$? If $N = 4$? If $N = 5$?
- (ii) What happens if $N = 50$?
- (iii) Can you describe what happens for a general N ?

2. There are N candies on a table. During each turn, a player must take one or two candies. The player who takes the last candy wins.

Questions:

- (i) Who has a winning strategy if $N = 2$? If $N = 3$? If $N = 4$? If $N = 5$?
- (ii) What happens if $N = 50$?
- (iii) Can you describe what happens for a general N ?

3. There are N candies on a table. During each turn, a player must take one, two, or three candies. The player who takes the last candy wins.

Questions:

- (i) Who has a winning strategy if $N = 2$? If $N = 3$? If $N = 4$? If $N = 5$?
- (ii) What happens if $N = 50$?
- (iii) Can you describe what happens for a general N ?

4. There are N candies on a table. During each turn, a player must take any number of candies as long as it is strictly less than half the candies on the table; or just one. The player who takes the last candy wins.

Questions:

- (i) Who has a winning strategy if $N = 2$? If $N = 3$? If $N = 4$? If $N = 5$?
- (ii) What happens if $N = 50$?
- (iii) Can you describe what happens for a general N ?