San Francisco Math Circle — Block 2, Fall 2007

The San Francisco Math Circle and Circle for Teachers are programs of the Mathematical Sciences Research Institute funded by the Moody's Foundation and S. D. Bechtel, Jr. Foundation.

Zome Challenges

1 *Metazomes*. Build a model of the zome ball using zomes! All you need are blue struts (small and medium or medium and long).

2 *Duality of Icosahedron and Dodecahedron*.

(a) Build a "red starburst" by putting small red popsicles into all the pentagonal holes of a single ball. The balls on these popsicles should form the vertices of an icosahedron. Complete the icosahedron with small blues.

(b) Then go back to your center ball, and add medium yellow popsicles to it. What do the balls of these popsicles make? Complete the construction with small blues. Explain what is going on.

3 *Fun with Greens*.

(a) Build a cube out of small blues. Then join some of the vertices of this cube to make a *tetrahedron*, a polyhedron consisting of four equilateral triangles, meeting three triangles to a vertex. The edges will be medium greens.

(b) Now make an *octahedron* using green edges. Start by making a blue "scaffolding" consisting of a ball (the center) with six blue edges emanating in the six perpendicular directions (east, west, north, south, up, down). The octahedron consists of 8 equilateral triangles, meeting four triangles to a vertex.

(c) Given two polyhedra all of whose edges have the same length: a pyramid with a square base, and a tetrahedron. Suppose we glue the two polyhedra together along a triangular face (so that the attached faces exactly overlap). How many faces does the new solid have? You may first argue that the pyramid has 5 faces and the tetrahedron has 4, so the combined polyhedron should have 7 faces ($5+4-2$ to account for the two interior faces). But this is wrong. What is the correct answer? Use greens!

4 *More Duality*. As you did in problem 2, now show that the octahedron and the cube are duals, and that the tetrahedron is dual to itself.

5 *Hidden in polyhedra*. Most people don't know about the following wonderful phenomena. Discover for yourself the

(a) tetrahedra hidden naturally in the cube (you will need green struts for this)

(b) golden rectangles in the icosahedron

(c) cubes in the dodecahedron

6 *A Space-Filling Polyhedron*. The tetrakaidecahedron has 14 faces: six squares and 8 regular hexagons. However, it is not possible to make a zome tetrakaidecahdron with all sides equal. Instead, we can make one where the squares are rectangles (two blues, two reds) and the two of the hexagons are regular (all blue) and four are made with 4 reds and 2 blues. Build this nice shape. The amazing thing about it is that it is possible to fill space with identical copies of it, with no overlap and no gaps! Show that this is so.

7 *Soccer Balls and Other Truncations*. If you "snip" the corners of a polyhedron, you get a new one, a *truncation* of the original. There are many ways to do truncations, but for now, try experimenting with two kinds: truncating by thirds, and truncating by halves. When you truncate by thirds, you go one third of the length of an edge away from the vertex, and "snip" there. Truncation by halves is the same idea, only now you snip at the halfway point. To do such truncations with zomes, you should first build a polyhedron that is magnified. For example, to do a truncation-by-halves, start with a polyhedron made with *double* strut lengths. For truncation-by-thirds, use triple lengths. Then it is easy to see where the snipping happens.

Try truncating the following.

(a) Cube (by halves)

(b) Cube (by thirds)

(c) Icosahedron (by thirds)

(d) Icosahedron (by halves)

(e) Dodecadedron (by halves)