Preliminary Schedule, MSRI Geometry and Representation Theory Workshop

Numbers in JML/JRM talks refer to The Geometry of Tensors book.

Week 1:

Monday 7/7

- **9-10am JML**: Complexity of matrix multiplication, an overview of Ch. 2 including tensors, rank and border rank of tensors, and wiring diagrams.
- **10:15-11:15am JRM:** Algebraic varieties § 3.1, 3.2. Basic definitions from algebraic geometry: projective space, variety, ideal, Zariski topology. Segre, Veronese, and other examples of varieties. Graphical models and motivating examples in statistics and information theory.
- **11:30am-12:30pm Lek-Heng:** Tensor approximations
- **Lunch**
- **2-5pm:** TA's cover problems from chap 2 (long problem session!)
- 5-5:30pm: TA's and lecturers meeting

Tuesday 7/8

- **9-10am JRM:** Finish Ch. 2 (§ 2.8, 2.9): decomposing V^{\ot 3}., G-modules, isotypic components. § 4.1,2 Representations, Schur's Lemma, G-modules and decomposing spaces of tensors
- **10:15-11:15am JML:** § 3.3,4,5,7 Tangent spaces to varieties, joins, cones, secant varieties, their dimension. The geometric definition of border rank, projective second fundamental form.
- **11:30am-12:30pm Vin**: Notions of tensor ranks: rank, border rank, multilinear rank, nonnegative rank
- **2-3pm** D. Gross lecture "What is quantum information theory?"
- **3:30-5pm** TA's cover problems
- 5-5:30pm: TA's and lecturers meeting

Notes: Jason needs to leave early on 7/8.

Wednesday 7/9

- **9-10am JML**: Finish Chap 3 - Terracini's lemma and applications to computing the dimension of secant varieties.
- **10:15-11:15am JRM:** § 4.3,4,5 - Representations of the symmetric group, Young diagrams, Young symmetrizers and wiring diagrams. Using these tools to decompose V^{\otimes d} as a GL(V) module. Schur-Weyl Duality.
- **11:30am-12:30pm Lek-Heng:** Conditioning, computations, applications
- **Afternoon:** Tilden Park BBQ

Thursday 7/10

- **9-10am JRM:** Toric varieties, toric ideals, moment map, exponential families.
- **10:15-11:15am JML:** § 4.6,7,8 Highest weight vectors, bases of highest weight space. Ideals of Segre, Veronese varieties and homogeneous varieties in general, decomposing S^d(A_1\otimes \cdots \otimes A_n), characters.
- **11:30am-12:30pm Vin:** Constructibility of the set of tensors of a given rank
- **2-3pm** L. Garcia lecture
- **3:30-5pm** TA's cover problems
- 5-5:30pm: TA's and lecturers meeting

Notes: Lek-Heng will leave at lunch and be gone Friday

Friday 7/11

- **9-10am JML:** § 5.1-5.3 Equations for secant varieties I: special Segre varieties, subspace varieties, flattenings,
- **10:15-11:15am JRM:** finish Ch 4 (Littlewood-Richardson rule and other handy formulas, more decompositions of spaces of tensors)
- **11:30am-12:30pm Vin:** Hyperdeterminants and optimal approximability
- **2-3pm** D. Gross lecture "What are graph states?"
- **3:30-5pm** TA's cover problems
- 5-5:30pm: TA's and lecturers meeting

Week 2:

Monday 7/14

- **9-10am JRM:** § 5.4, 5.5 Equations II: inheritance, and prolongation
- **10:15-11:15am JML:** § 5.6 Equations III: Strassen's equations and variants
- **11:30am-12:30pm Vin:** Uniqueness of tensor decomposition, direct sum conjecture
- **2-3pm** Risi Kondor lecture "A complete set of rotationally and translationally invariant features based on a generalization of the bispectrum to non-commutative groups"
- **3:30-5pm** TA's cover problems
- 5-5:30pm: TA's and lecturers meeting

Tuesday 7/15

- **9-10am JML:** § 6.1,6.2,6.6,6.7 The Alexander-Hirshowitz theorem and dimensions of secant varieties of Segre varieties
- **10:15-11:15am JRM:** Ch 7. An algorithm for explicitly writing down polynomials in a given submodule of the space of polynomials. Further combinatorics of Young tableaux. Working with tensors in factored vs. expanded form.
- **11:30am-12:30pm Lek:** Nonnegative hypermatrices, symmetric tensors
- **2-3pm** Luke Oeding lecture "The variety of principal minors of symmetric matrices"
- **3:30-5pm** TA's cover problems
- 5-5:30pm: TA's and lecturers meeting

Wednesday 7/16

- **9-10am** Comon: (a) general statements on linear mixtures of random variables, (b) cumulants, (c) tensors
- **10:15-11:15am** Weyman lecture I:What do the words "ACM", "Gorenstein", and " rational singularites" mean and why are these properties useful?
- **11:30am-12:30pm JML:** Ch 8: Rank vs border rank of tensors and symmetric tensors
- **Afternoon:** free

Thursday 7/17

- **9-10am** Comon: (d) the invertible case: Independent Component Analysis - optimization criteria and some numerical algorithms
- **10:15-11:15am** Weyman lecture II: Introduction to the study of G-varieties via desingularizations by homogeneous vector bundles
- **11:30am-12:30pm JML:** Ch 9: Spaces of tensors admitting normal forms
- **2-3:30pm:** TA's cover last homeworks
- **4-5pm**: Student lecture TBA

Friday 7/18

- **9-10am** Comon: (e) the UDM case: some selected statistical blind identification approaches, all involving tensors. Local identifiability and numerical algorithms (including BIOME and FOOBI).
- **10:15-11:15am** Ottaviani lecture I: Induction for the rank of tensors
- **11:30am-12:30pm** student lecture TBA or extra course lecture
- **2-3pm** Ottaviani lecture II: The Alexander-Hirschowitz theorem
- **3:30-5pm** Student lecture TBA

Abstracts of guest lectures:

G. Ottaviani:

I: Induction for the rank of tensors
Abstract: we describe an inductive procedure which allows to compute in many cases the generic rank of tensors, which can be seen as multidimensional matrices. In the hypercubic case we get a bound which is asymptotically sharp. This is joint work with H. Abo and C. Peterson.

II:The Alexander-Hirschowitz theorem
Abstract: The Alexander-Hirschowitz theorem says that the secant varieties of the Veronese varieties have the expected dimension, with a well known list of exceptions. The theorem has several reformulations, in the setting of polynomial interpolation or in the setting of rank of symmetric tensors. We discuss the proof of the theorem, which is by induction. Both the inductive step and the starting case of the induction (the cubic case) have some interesting feature.

Tensor problems in Engineering
P.Comon, MSRI, 16-18 July, 2008
In Engineering, the identification of a linear statistical model is omnipresent (antenna array processing, factor analysis, etc). This problem has been extensively addressed since the fifties, refer e.g. to early contributions of Darmois and Skitovich. It consists of estimating a mixing matrix from observed realizations, under the assumption that the unobserved source random variables exciting the model are statistically independent.
However, existence and uniqueness theorems were generally not constructive, in the sense that they did not yield practical algorithms. It is shown how such problems can be stated in terms of the decomposition of symmetric tensors into a sum of rank-1 terms. For instance, Cumulant tensors have been used in different Engineering problems, and are symmetric as high-order derivatives of a multivariate scalar function.
Various numerical algorithms are surveyed, and actually attempt to find a compromise between sub-optimality, numerical complexity, and simplicity. Principles underlying these algorithms are outlined, depending on the type of mixing matrix in the model: orthogonal, square invertible, or rectangular with full row rank. The case of square mixtures leads to approximate decompositions, whereas rectangular mixtures allow exact decompositions of so-called Under-determined Mixtures (UDM). It is pointed out during the course that some problems remain open.

In some cases, the observation model itself is multi-linear, which may avoid to compute tensors (e.g. cumulants) from data. In that case, solutions become deterministic, and tensors to be decomposed generally do not enjoy symmetries. Whereas the courses given at MSRI focus only on statistical approaches, the AIM/ARCC talk also covers deterministic approaches.