Exercises for Lecture 1.
E. Kerman

1. Let $x\colon S^1 \to M$ be a nondegenerate 1-periodic orbit of a Hamiltonian vector field X_H on (M, ω). Show that x is isolated away from all other 1-periodic orbits.

2. Find a function $f\colon \mathbb{R}^{2n} \to \mathbb{R}$ such that the origin is a nondegenerate critical point of f but not (the image of) a nondegenerate periodic orbit.

3. Let Λ be the space of smooth maps from S^1 to M which are contractible, i.e., homotopic to constant maps. For $x \in \Lambda$, the tangent space $T_x\Lambda$ is the space of smooth sections ξ of $(TM)|_{x(S^1)}$.

   (a) Consider the action functional $\mathcal{A}_H\colon \Lambda \to \mathbb{R}$ defined by

   $$\mathcal{A}_H(x) = \int_{S^1} H(t, x(t))\, dt - \int_{D^2} \bar{x}^*\omega,$$

   where (again) $\bar{x}$ is a smooth map from the unit disc $D^2 \subset \mathbb{C}$ to M such that $\bar{x}(e^{2\pi i t}) = x(t)$. Compute $(d\mathcal{A}_H)_x(\xi)$.

   (b) Let J_t is an S^1-family of almost complex structures. J_t determines a metric on Λ as follows:

   $$\langle \xi, \eta \rangle = \int_0^1 \omega(\xi(t), J_t\eta(t)),\, dt.$$

   Show that the negative gradient trajectories of $\mathcal{A}_H$ with respect to the metric determined by J_t are Floer trajectories.