

## EXERCISES FOR MSRI GRAD WORKSHOP

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### 1. LECTURE 1

- (1) For the left-hand trefoil, we have  $\overline{tb} = -6$  and  $\overline{sl} = -5$ . Find Legendrian and transverse representatives achieving these maxima.
- (2) Prove that  $\overline{tb}(K) \leq \overline{sl}(K)$  for all oriented topological knots  $K$ . (Beware:  $K$  is not necessarily the same as its orientation reverse!)
- (3) The pretzel knot  $P(-3, 5, 7)$  is famously topologically slice. Use slice-Bennequin to prove that it is not smoothly slice.

### 2. LECTURE 2

- (1) Prove that if a knot is Legendrian simple, then it is transversely simple.
- (2) Prove that the resolution of a front is the  $xy$  projection of a Legendrian knot Legendrian isotopic to the original.
- (3) Prove that  $\partial$  in Legendrian contact homology lowers degree by 1: for an immersed disk with boundary on the  $xy$  projection of  $L$ , positive corner at  $a_i$ , and negative corners at  $a_{j_1}, \dots, a_{j_k}$ ,

$$|a_i| \equiv |a_{j_1}| + \dots + |a_{j_k}| + 1 \pmod{2r(L)}.$$

### 3. LECTURE 3

- (1) Check: if  $(V, \alpha)$  is contact then  $d(e^t\alpha)$  is symplectic on  $\mathbb{R} \times V$ ; nontrivial critical points for the action functional  $\gamma \mapsto \int_\gamma \alpha$  on the free loop space of  $V$  are Reeb orbits.
- (2) Prove that the standard Legendrian right-hand trefoil (with  $tb = 1$ ) is not destabilizable, using contact homology rather than bounds on  $tb$ .
- (3) Prove that the Chekanov  $5_2$  knots are Legendrian isotopic after one stabilization of each (disregard orientation if you like). Hint: choose the placement of the zigzags carefully.