



Mathematical Sciences Research Institute

Analytical and Stochastic Fluid Dynamics

Organized by: Craig Evans, Susan Friedlander, Boris Rozovsky, Daniel Tataru and David A. Ellwood
October 10, 2005 to October 14, 2005

Lecture Schedule & Talk Abstracts

Monday, October 10

- 9:30-10:00 **Mohammed Ziane**, “Remarks on the normal form of the Navier-Stokes equations”
Abstract: We introduce a construction of regular solutions to the Navier-Stokes which is specifically designed for the study of their asymptotic expansions. Using this construction, we give sufficient conditions for the convergence of those expansions. We also construct suitable normed spaces in which they converge. Moreover, in these spaces, the normal form of the Navier-Stokes equations associated with the terms of the asymptotic expansions is a well-behaved infinite system of differential equations. This is a joint work with Foias, Hoang and Olson.
- 10:00-10:45 **Peter Constantin**, “Nonlinear Fokker-Planck Navier-Stokes Systems”
Abstract: We will describe regularity results concerning Navier-Stokes equations coupled with nonlinear Fokker-Planck equations describing the probability distribution of microscopic inclusions.
- 10:45-11:15 Morning Tea (6th Floor)
- 11:15-12:00 **Edriss Titi**, “Global Regularity for the Three-dimensional Primitive Equations of Ocean and Atmosphere Dynamics”
Abstract: The basic problem faced in geophysical fluid dynamics is that a mathematical description based only on fundamental physical principles, which are called the “Primitive Equations”, is often prohibitively expensive computationally, and hard to study analytically. In this talk we will survey the main obstacles in proving the global regularity for the three-dimensional Navier–Stokes equations and their geophysical counterparts. Even though the Primitive Equations look as if they are more difficult to study than the three-dimensional Navier–Stokes equations we will show in this talk that they have globally (in time) unique regular solution for all initial data. This is a joint work with Chongshen Cao.
- 12:00-2:45 Lunch/Discussion
- 2:45-3:30 **Sergei B. Kuksin**, “Asymptotic properties of some SPDE with small dissipation”
Abstract: I will discuss recent progress in the study of asymptotic properties of a class of randomly forced PDE, where the force is proportional to the square root of the dissipation. The class contains, for example, the randomly forced 2D Navier-Stokes equation and the forced-driven KdV equation. The latter equation will be the main object of my talk.
- 3:30-4:00 Afternoon Tea (6th Floor)
- 4:00-4:45 **Franco Flandoli**, “Markov selections and their regularity for 3D stochastic Navier-Stokes equations”
Abstract: Existence of weak solutions to the 3D stochastic Navier-Stokes equations in the martingale sense is known, but their uniqueness is open as in the deterministic case. We prove the existence of selections with a weak form of Markov property. Under special assumptions on the noise, we prove the full Markov property and a form of continuous dependence on initial conditions, similar to the Strong Feller property. The methods are strongly inspired to an approach of Da Prato and Debussche.
- 4:45-5:15 **Roman Shvydkoy**, “Spectral problem for the Euler and Navier-Stokes equations”
Abstract: In this talk we give an overview of recent results on the essential spectrum of the Euler equation, related instability issues, and the vanishing viscosity limit for the linearized problem.

Tuesday, October 11

9:30-10:00 **Eric Vanden-Eijnden**, "Simple Solvable Models with Cascade of Energy and Anomalous Dissipation"
Abstract: We will present a class of simple and exactly solvable linear shell models which are formally conservative yet may display anomalous dissipation via cascade of energy. As a result, these models can be forced and yet have a unique invariant measure.

10:00-10:45 **Jonathan Mattingly**, "Exponential Mixing for the Degenerately forced Navier Stokes Equations"
Abstract: I will describe how to prove exponential convergence to the unique equilibrium of the stochastically forced 2D Navier-Stokes equations when the forcing is minimal. The techniques are quite general and make use of infinite dimensional Malliavin calculus and coupling in the Wasserstein Metric on a function space.

10:45-11:15 Morning Tea (6th Floor)

11:15-12:00 **Vladimir Sverak**, "Regularity of L^3 solutions of the Navier-Stokes equations"
Abstract: This is a joint work with Luis Escauriaza and Gregory Seregin showing that solutions of the three-dimensional incompressible Navier-Stokes equations with bounded spatial L^3 norm are regular. The proof uses a blow-up technique, together with a backward uniqueness result for the heat equation.

12:00-2:45 Lunch/Discussion

2:45-3:30 **Charlie Fefferman**, "The surface QG-alpha equation"
Abstract: Will explain recent work of Cordoba et al on a 2D fluid equation intermediate between surface QG and 2D Euler.

3:30-4:00 Afternoon Tea (6th Floor)

4:00-4:45 **Tom Hou**, "The Interplay between Local Geometric Properties and the Global Regularity for 3D Incompressible Flows"
Abstract: Whether the 3D incompressible Euler equation can develop a finite time singularity from smooth initial data has been an outstanding open problem. It has been believed that a finite singularity of the 3D Euler equation could be the onset of turbulence. Here we review some existing computational and theoretical work on possible finite blow-up of the 3D Euler equation. Further, we show that there is a sharp relationship between the geometric properties of the vortex filament and the maximum vortex stretching. By exploring this local geometric property of the vorticity field, we have obtained a global existence of the 3D incompressible Euler equations provided that the unit vorticity vector and the velocity field have certain mild regularity property in a very localized region containing the maximum vorticity. Our assumption on the local geometric regularity of the vorticity field and the velocity field seems consistent with recent numerical experiments. Further, we discuss how viscosity may help preventing singularity formation for the 3D Navier-Stokes equations, and prove the global existence of the 2D dissipative Boussinesq equations. We will also present some recent global existence results for a model of the 3D Navier-Stokes equations and the 3D alpha-averaged Euler equations.

4:45-5:15 **Jim Kelliher**, "Bounded domain limit for Navier-Stokes and Euler equations"
Abstract: Suppose we have well-posedness for some PDE both in all of R^d and in a bounded domain U of R^d . Imagine we take the initial data for the whole space and restrict it to U , modifying it slightly to satisfy any required boundary conditions. If we scale U by r , does the solution on rU approach the solution in R^d as r goes to infinity in some appropriate sense?

We answer this question for weak solutions to the Navier-Stokes and Euler equations in two dimensions, showing that strong convergence occurs in some norms and weak convergence in other norms of interest. Critical to this is showing that the initial velocity can be modified to match the appropriate boundary conditions while changing its norms by a vanishingly small amount as r goes to infinity. A secondary goal is to establish, for initial velocity that decays at infinity as slowly as possible, the same existence, uniqueness, and regularity results as for classical (that is, finite energy) solutions.

The original motivation for this work was to establish the existence and uniqueness of statistical solutions to the Navier-Stokes and Euler equations in the entire plane with infinite energy and, for the Euler

equations, unbounded initial vorticities, by taking advantage of the well developed theory of statistical solutions in a bounded domain of the plane.

Wednesday, October 12

- 9:30-10:00 **Igor Kukavica**, “One direction and one component regularity for the Navier-Stokes equations”
Abstract: We consider sufficient conditions for regularity of Leray-Hopf solutions of the Navier-Stokes equation. By a result of Neustupa and Panel, the Leray-Hopf weak solutions are regular provided a single component of the velocity is bounded. In this talk we will survey existing and present new results on one component and one direction regularity. This is a joint work with M. Ziane
- 10:00-10:45 **Andrea Bertozzi**, “Electrowetting in a Hele-Shaw geometry”
Abstract: Electrowetting has recently been explored as a mechanism for moving small amounts of fluid in confined spaces. We propose a diffuse interface model for droplet motion, due to electrowetting, in a Hele-Shaw geometry. In the limit of small interface thickness, asymptotic analysis shows the model is equivalent to Hele-Shaw flow with a voltage-modified Young-Laplace boundary condition on the free surface. We show that details of the contact angle significantly affect the timescale of motion in the model. We measure receding and advancing contact angles in the experiments and derive their influences through a reduced order model. These measurements suggest a range timescales in the Hele-Shaw model which include those observed in the experiment. The shape dynamics and topology changes in the model agree well with the experiment, down to the length scale of the diffuse interface thickness. This is joint work with Hsiang-Wei Lu, Karl Glasner, and C J Kim.
- 10:45-11:15 Morning Tea (6th Floor)
- 11:15-12:00 **Giovanni Gallavotti**, “Chaotic motions and developed turbulence: heuristic ideas”
Abstract: An interpretation of Ruelle's proposal to regard turbulence as due to strange attractors can be interpreted as the statement that, for practical purposes, the attractor of a chaotic evolution can be regarded as an Anosov system. The problem is how to extract information that can be tested experimentally or theoretically from this point of view. I shall discuss some recent attempts in this direction.
- 12:00-2:45 Lunch/Discussion
- 2:45-3:30 **Claude Bardos**, “Analytic stability and singularities for Kelvin Helmholtz, Rayleigh Taylor, Problems Comparison with the stability of water waves problems”
Abstract: Rayleigh Taylor and Kelvin Helmholtz are equations describing interfaces in fluids. Such interfaces are known by experiments and numerical simulations to be highly unstable. On the other hand a series of recent of results (Sijie Wu, Gilles Lebeau and Vladimir Kamotski) show that these solutions whenever they exist with a minimal regularity are solutions of nonlinear elliptic equations. Therefore they are analytic. This shows for instance that any solution with small regularity (say in $C^{l+\infty}$ but not in C^∞) at some time will evolve in very singular systems. In the present lecture I will discuss these issues and compare the situation corresponding to Kelvin Helmholtz, Rayleigh Taylor and water waves equations. Conclusions are as follow The equation for water waves describes the stability of the phenomena as long as the surface of the water (which may be a multivalued function) do not self intersect. From this point of view Rayleigh Taylor and Kelvin Helmholtz problem are very similar but very different from water waves. When a singularity appears say a cusp in vorticity as predicted by the numerical computations of Moore and Orzag and proven on one example by Caflisch and Orellana the solution can in no way be extended by a curve of the same regularity what will appear will be a very weak solution in the sense of Delors or a curve which will not be arc-chord in the sense of Guy David and may be in agreement with the numerical simulations of Krasny. Therefore mathematical proof of the existence of weak enough solutions concentrated on such curves is a challenging open problem.
- 3:30-4:00 Afternoon Tea (6th Floor)
- 4:00-4:45 **Herbert Koch**, “Regularity for a free boundary problem and a conjecture of De Giorgi”
Abstract: In this joint work with G. Leoni and M. Morini we transform free boundary problems to systems of elliptic boundary value problems. Examples are minimizers to the Mumford-Shah functional and stationary velocity fields of fluids with surface tension. Koch: Regularity for a free boundary problem and a conjecture of De Giorgi.

4:45-5:15 **Natasa Pavlovic**, “Long time behavior of solutions to the 3D Navier-Stokes”
Abstract: In this talk we shall discuss long time behavior of solutions to the 3D Navier-Stokes equations that evolve from initial data in the space BMO^{-1} intersected with certain Morrey space. This is a joint work with James Colliander, Carlos Kenig and Gigliola Staffilani.

Local in time existence of solutions to the Navier-Stokes equations in BMO^{-1} and global in time existence of solutions corresponding to small initial data in BMO^{-1} were proved by Koch and Tataru in 2001. We consider a priori global solution in the space BMO^{-1} corresponding to initial data that are in BMO^{-1} intersected with certain Morrey space. A persistence of Morrey norm combined with microlocal analysis of solutions help us identify a time decay of solutions.

Thursday, October 13

9:30-10:00 **Anna L. Mazzucato**, “On the decay of the energy spectrum for weak solutions to the Navier-Stokes equations”

Abstract: We observe that known regularity results imply weak solutions of the 3D Navier-Stokes equations are regular if their energy spectrum decays as k^s , $s < -3$, for large wavenumbers k . We allow for weak solutions having infinite total energy by localizing the equations. We then consider certain modified Leray backwards self-similar solutions and show that their energy spectrum decays exactly at the critical rate. Therefore, this rate of decay is consistent with the appearance of a self-similar singularity.

10:00-10:45 **Anatoli Babin**, “Linear superposition of nonlinear waves”

Abstract: Nonlinear waves are described by nonlinear differential equations. Their solutions are determined by initial data, which are functions of the spatial variables. When the equation is linear, if the initial function equals the sum of two or several functions, the solution equals the sum of corresponding solutions. For nonlinear equations this linear superposition principle is not valid. Nevertheless, there are important and physically relevant systems and classes of initial data for which the solution equals the sum of corresponding solutions with a small error. The examples include: Fermi-Pasta-Ulam system, nonlinear wave equation, nonlinear Schrodinger equation, Navier-Stokes and Euler systems in a rotating frame, Boussinesq system with a strong rotation or stratification. The described approximate linear superposition of nonlinear waves is explained by a destructive wave interference between different wavepackets in the process of their time evolution, this interference drastically reduces nonlinear interactions between the wavepackets.

10:45-11:15 Morning Tea (6th Floor)

11:15-12:00 **Susan Friedlander**, “Nonlinear Instability for the Navier Stokes Equations”

Abstract: It is proved that linear instability implies nonlinear instability for the Navier Stokes equations in L^p , $p > 1$. The result holds in all spatial dimensions and both finite domains and R^n . The method of proof uses a bootstrap argument. This is joint work with Roman Shvydkoy and Natasa Pavlovic. Friedlander: Nonlinear Instability for the Navier Stokes Equations

12:00-2:45 Lunch/Discussion

2:45-3:30 **Poster Session**

3:30-4:00 Afternoon Tea (6th Floor)

4:00-4:45 **Poster Session** (continued)

4:45-5:15 **Discussion Workshop**

SUBMITTED POSTERS:

Nirmalendu Chaudhuri

"Scaling Property of Thin Martensitic Films"

Helena J. Nussenzeig Lopes

"Planar incompressible viscous flow around a small obstacle"

Friday, October 14

9:30-10:00 **Boris Rozovsky**, “Passive Scalar Equation in a Turbulent Gaussian Velocity Field”

Abstract: Time evolution of a passive scalar is considered in a turbulent homogeneous incompressible Gaussian flow. The turbulent nature of the flow results in non-smooth coefficients in the corresponding evolution equation. A strong, in the probabilistic sense, solution of the equation is constructed using Wiener Chaos expansion, and the properties of the solution are studied. Among the results obtained are a certain

L_p -regularity of the solution and Feynman-Kac-type, or Lagrangian, representation formula. The results apply to both viscous and conservative flows. Rozovsky: Passive Scalar Equation in a Turbulent Gaussian Velocity Field.

- 10:00-10:45 **Alexandre Chorin**, “Scaling laws in turbulence”
Abstract: I will present a mathematical model for the scaling of wall-bounded turbulent flows, as well as recent results on the scaling of homogeneous isotropic turbulence. (joint work with G.I. Barenblatt).
- 10:45-11:15 Morning Tea (6th Floor)
- 11:15-12:00 **Remigijus Mikulevicius**, “On stochastic Euler equation”
Abstract: The existence of a martingale problem solution to a degenerated Navier-Stokes equation is proved. It is a weak limit of solutions to Navier-Stokes equation.
- 12:00-2:45 Lunch/Discussion
- 2:45-3:30 **Xiaoming Wang**, “The Emergence of Large Scale Coherent Structure under Small Scale Random Bombardments”
Abstract: We provide mathematical justification of the emergence of large scale coherent structure in a two dimensional fluid system under small scale random bombardments with small forcing and appropriate scaling assumptions. The analysis shows that the large scale structure emerging out of the small scale random forcing is not the one predicted by equilibrium statistical mechanics. But the error is very small which explains earlier successful prediction of the large scale structure based on equilibrium statistical mechanics. This is a joint work with Andrew Majda of New York University.
- 3:30-4:00 Afternoon Tea (6th Floor)