

This week-long workshop will feature expository talks giving the “big picture” regarding the rational and integral solutions to systems of multivariable polynomial equations. When the variety defined by the equations is of dimension 1, much is known, both in theory and in practice, so the focus of the workshop (and of the semester) will be on the case of dimension  $> 1$ , where most of the basic questions are far from being answered, and there is great opportunity for new results.

The talks will be at a level appropriate for mathematicians outside the field, and for postdocs and graduate students entering the field. Each will give an overview of an aspect of the field, with an eye towards providing the background needed to understand areas of current research and their associated open problems.

Some open questions to be discussed include:

- For which varieties should one expect there to be few or no rational points? To what extent can the answer be read from the geometry of the associated complex manifold? Can cohomology explain the non-existence of rational points on each variety that has no rational points?
- For varieties with infinitely many rational points, can one estimate the number of such points whose coordinates have numerator and denominator bounded by  $B$ , as a function of  $B$  as  $B \rightarrow \infty$ ? To what extent can analytic methods help answer this question?
- For a fixed elliptic curve (or more generally, abelian variety), it is known that the group of rational points is finitely generated. How does the rank of this group vary in a family of such varieties depending on a parameter?
- It is known that there is no algorithm for deciding whether a system of polynomial equations has solutions where the variables take values in  $\mathbb{Z}$ . Can one prove an analogous undecidability result for  $\mathbb{Q}$ ?