APPLICATIONS OF GROUPS AND ISOMORPHIC GROUPS TO TOPICS IN THE STANDARD CURRICULUM, GRADES 9–11: PART I

Many relationships between groups and topics of secondary school mathematics are shown by the author, who proposes that the study of groups be included as standard fare in the mathematics curriculum of the average college-bound student.

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THE subfield of pure mathematics that has grown most significantly in the past few decades is that of algebra, by which is meant "higher" or "abstract" algebra and linear algebra. Twenty years ago courses in algebra were at the advanced undergraduate and graduate level, and it was easy to become a certified mathematics teacher without having any knowledge of groups, rings, fields, or vector spaces. Today virtually all prospective teachers take a course in which some of these structures are studied.

Yet, if we judge the situation from our textbooks, we find most students who take eleven years of school mathematics rarely explicitly encounter any of the common algebraic structures except by way of discussions of field properties. Even though such material often appears early in texts and is seemingly necessary for future work, the ideas are seldom used in any constructive way.

At the present time, in the United States, only books written for *bright* students have used structures as an integral part of the development. Two examples are *A Vector Approach to Euclidean Geometry,* by Vaughan and Szabo (New York: Macmillan, 1972) and *Unified Mathematics,* vols. 1–3, by Fehr, Fey, and Hill (Reading, Mass.: Addison-Wesley Publishing Co., 1972). Yet in foreign texts, groups and other structures are not uncommon even for average students. For example, see the texts of the School Mathematics Project (New York: Cuisenaire Co.) and *Modern Mathematics 1* and *2*, by Papy (New York: Macmillan Co., 1968, 1970).

The United States situation, in view of the importance that algebra now has in mathematics, may be serious. A recent survey found that 15% of the doctorates in mathematics in 1968–71 were in algebra, second only to analysis (21%) among the branches (*CBMS Newsletter,* vol. 7, no. 3, May 1972). However, it cannot be assumed that something ought to be studied by high school students just because it is important to mathematicians. But it can be assumed that algebraic structures should at least be given serious consideration. "Groups" are among the most fundamental of these structures and seem to be very amenable to early study by students. For example, several books by Dienes and Golding (1967 [a], [b], and [c]) contain activities for children in grades 3 through 6.

The major purpose of this article is to exhibit relationships between groups and various topics in secondary school mathematics. By presenting a wide variety of topics and approaches, it is hoped to add evidence to the increasingly strong case for the inclusion of some study using groups as standard fare in the mathe-