

# BAY AREA DIFFERENTIAL GEOMETRY SEMINAR

Saturday, April 17, 2010

MSRI, Berkeley

10:30	<b>Coffee</b>
11:00 - 12:00	<p>▶ <b>Robert Bryant</b>—MSRI and UC-Berkeley</p> <p><b>TITLE:</b> Classification of local conformally flat Ricci solitons</p> <p><b>ABSTRACT:</b> Recently, there has been work that classifies conformally flat gradient Ricci solitons under the assumptions of completeness and bounded curvature in the steady and shrinking cases. The known results are that, under these assumptions, the soliton must be rotationally symmetric, and the proofs make essential use of global machinery and estimates that do not hold for local Ricci solitons.</p> <p>In this talk, I will show that, in dimension 3 and higher, any <i>local</i> conformally flat gradient Ricci soliton is locally rotationally symmetric in a slightly generalized sense (one that allows for flat or hyperbolic rotational symmetry in addition to the usual spherical (i.e., elliptic) rotational symmetry). In addition, I will report on my (so far) partial results classifying conformally flat Ricci solitons without the assumption that they be of gradient type.</p>
12:00 - 2:00	<b>Lunch and organizational meeting</b>
2:00 - 3:00	<p>▶ <b>Xianzhe Dai</b>—UC-Santa Barbara</p> <p><b>TITLE:</b> Intersection R-torsion and analytic torsion for manifolds with conical singularity</p> <p><b>ABSTRACT:</b> The R-torsion, introduced by Reidemeister in 1935, is the first topological invariant that are not homotopy invariant.</p> <p>The analytic torsion is introduced by Ray-Singer in the '70s as an analytic analogue of R-torsion. The famous Ray-Singer conjecture says that these two are indeed equal on closed manifolds. The Ray-Singer conjecture was proved independently by Cheeger and Mueller in the late '70s. Now both the R-torsion and analytic torsion can be generalized to singular manifolds, at least for manifolds with conical singularity. It is thus an intriguing question whether and how the Ray-Singer conjecture generalize to this setting. We will discuss some of the recent work in this direction.</p>
3:00 - 3:30	<b>Break</b>

3:30 -  
4:30

► **Masood Ul-Alam**—City College of San Francisco

**TITLE:** Proof that static stellar models are spherical

**ABSTRACT:** We outline the proof that static general-relativistic stellar models, in other words static perfect fluid stars in asymptotically flat space-times, are spherically symmetric for physically reasonable equations of state relating the pressure and the density of the fluid.

For a static Newtonian star the metric of the 3-space is Euclidean. In the absence of shape-maintaining forces, gravitation levels off "mountains," making the static star spherical. In the general-relativistic case the metric is determined by Einstein equations and it should not be assumed spherically symmetric without a proof. The problem is essentially as follows.

Suppose that  $(\Sigma, g)$  is an asymptotically flat 3-dimensional Riemannian manifold and  $V$  is a positive function on  $\Sigma$ . Suppose further that  $V$  and  $g$  satisfy the equations

$$R_{ij} = V^{-1}V_{;ij} + 4\pi(\rho - p)g_{ij}, \Delta V = 4\pi V(\rho + 3p)$$

Here  $R_{ij}$  is the Ricci curvature of  $g$ .  $\Delta$  denotes the Laplacian, and  $;$  denotes covariant derivative relative to  $g$ . The density function  $\rho$  and the pressure function  $p$  vanish outside a compact set. Asymptotic conditions are

$$V = 1 - \frac{M}{r} + O(r^{-2}), g_{ab} = (1 + \frac{2M}{r})\delta_{ab} + O(r^{-2})$$

where the constant  $M$  is the mass,  $\delta_{ab}$  is the Euclidean metric, and  $r$  is a spherical coordinate associated with  $\delta_{ab}$ .

Then, assuming physically reasonable conditions on  $\rho$  and  $p$ , we show that, in some spherical coordinate system  $\{r, \theta, \varphi\}$ ,  $V = V(r)$  and  $g$  is of the form

$$g = g_{rr}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

It appears that the problem does not allow modifications of the symmetry proofs for the Poisson Equation in Euclidean space and more complicated systems considered by Gidas, Ni, and Nirenberg. Our proof uses a positive mass type argument. It was published in "General Relativity Gravitation," **39**, pp. 55-85 (2007) and is available online at

<http://www.springerlink.com/content/2723v381743g32x0>

**Saturday evening:** Banquet at 6:00pm—David Hoffmann's house