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title: Markov chains on Young diagrams.

problem:

Given a partition λ of number n into positive integers $n = \lambda_1 + \lambda_2 + \cdots + \lambda_k$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq 0$ we draw it as a Young diagram, which is a collection of unit boxes arranged in rows. In the first row there are λ_1 boxes, in the second - λ_2 and so on.

Let $\mathbb{Y}_n$ denote the set of all Young diagrams with n boxes. The sets $\mathbb{Y}_n$ have a natural inductive structure, namely, you can construct any Young diagram λ by starting from the empty diagram and then adding boxes one after another.

Now let U_n denote the uniform measure on $\mathbb{Y}_n$. The question is whether measures U_n have the same inductive structure as the sets $\mathbb{Y}_n$.

More formally, does there exist a Markov chain $X(n)$, $n = 1, 2, 3, \ldots$ such that for every n

1. $X(n) \in \mathbb{Y}_n$ and the distribution of $X(n)$ is $U(n)$,
2. The set-theoretical difference $X(n+1) \setminus X(n)$ consists of one box almost surely?

As far as I know, the first person to ask this question was Anatoly Vershik.

background:

There are some beautiful results about the properties of measures U_n. For instance, U_n-distributed Young diagrams have a limit shape as $n \to \infty$.

One of the most known non-uniform measures on the Young diagrams is the Plancherel measure, which is a probability measure on $\mathbb{Y}_n$ assigning to the Young diagram λ the weight $dim^2(\lambda)/n!$. Here $dim(\lambda)$ is the *dimension* of λ which can be defined combinatorially as the number of ways to grow a Young diagram λ out of empty diagram by adding boxes. $dim(\lambda)$ coincides with the dimension of the irreducible representation of symmetric group $S(n)$ indexed by λ, and there are also some explicit formulas for $dim(\lambda)$. Plancherel measure have some similarities with the uniform one. For instance, the limit shape phenomena also holds for it. However, the representation-theoretic origin of the Plancherel measure immediately implies the existence of the desired Markov chain for it, and this chain helped a lot in studying the measure. Transitional probabilities of the chain are

$$P(\lambda \to \mu) = \frac{dim(\mu)}{(n+1)dim(\lambda)}, \quad \mu \in \mathbb{Y}_{n+1},\ \lambda \in \mathbb{Y}_n.$$