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**title:** Markov chains on Young diagrams.

**problem:**

Given a partition  $\lambda$  of number  $n$  into positive integers  $n = \lambda_1 + \lambda_2 + \dots + \lambda_k$ ,  $\lambda_1 \geq \lambda_2 \geq \dots \geq 0$  we draw it as a Young diagram, which is a collection of unit boxes arranged in rows. In the first row there are  $\lambda_1$  boxes, in the second -  $\lambda_2$  and so on.

Let  $\mathbb{Y}_n$  denote the set of all Young diagrams with  $n$  boxes. The sets  $\mathbb{Y}_n$  have a natural inductive structure, namely, you can construct any Young diagram  $\lambda$  by starting from the empty diagram and then adding boxes one after another.

Now let  $U_n$  denote the uniform measure on  $\mathbb{Y}_n$ . The question is whether measures  $U_n$  have the same inductive structure as the sets  $\mathbb{Y}_n$ .

More formally, does there exist a Markov chain  $X(n)$ ,  $n = 1, 2, 3, \dots$  such that for every  $n$

1.  $X(n) \in \mathbb{Y}_n$  and the distribution of  $X(n)$  is  $U(n)$ ,
2. The set-theoretical difference  $X(n+1) \setminus X(n)$  consists of one box almost surely?

As far as I know, the first person to ask this question was Anatoly Vershik.

**background:**

There are some beautiful results about the properties of measures  $U_n$ . For instance,  $U_n$ -distributed Young diagrams have a limit shape as  $n \rightarrow \infty$ .

One of the most known non-uniform measures on the Young diagrams is the Plancherel measure, which is a probability measure on  $\mathbb{Y}_n$  assigning to the Young diagram  $\lambda$  the weight  $dim^2(\lambda)/n!$ . Here  $dim(\lambda)$  is the *dimension* of  $\lambda$  which can be defined combinatorially as the number of ways to grow a Young diagram  $\lambda$  out of empty diagram by adding boxes.  $dim(\lambda)$  coincides with the dimension of the irreducible representation of symmetric group  $S(n)$  indexed by  $\lambda$ , and there are also some explicit formulas for  $dim(\lambda)$ . Plancherel measure have some similarities with the uniform one. For instance, the limit shape phenomena also holds for it. However, the representation-theoretic origin of the Plancherel measure immediately implies the existence of the desired Markov chain for it, and this chain helped a lot in studying the measure. Transitional probabilities of the chain are

$$P(\lambda \rightarrow \mu) = \frac{dim(\mu)}{(n+1)dim(\lambda)}, \quad \mu \in \mathbb{Y}_{n+1}, \lambda \in \mathbb{Y}_n.$$