

Summer Graduate Workshop: Geometric Measure Theory

July 11-22, 2011 – MSRI

Titles and Abstracts

1. Luigi Ambrosio

- Title: Elements of Geometric Measure Theory in metric spaces
- Abstract: In this course I will present the basic facts of the theory of currents from a relatively new viewpoint, suggested by De Giorgi and developed by me and Kirchheim in 1999-2000. This viewpoint emphasizes the metric structure and provides new proofs of the classical results by Federer and Fleming even in the Euclidean case, together with new rectifiability results based either on slices or on projections. If time allows, I will also cover Wenger's proof of the isoperimetric inequality in spaces with good filling properties.

2. Thierry DePauw

- Title: Integral Geometric Measure
- Abstract: The talks will introduce this measure in Euclidean space, as well as the Cauchy-Crofton formula and the Besicovitch-Federer-Mickle structure theorem.

3. Robert Hardt

- Title: Flat Chains
- Abstract: The flat norm (and flat distance) was defined by Whitney in 1957 to give a notion of geometric closeness of chains. For example, a moving point mass coming to rest converges in the flat norm distance (and also weak topology) but not in the mass norm distance. For currents (i.e. real chains) the flat norm is dual to a "flat norm" on differential forms. Currents with finite mass and finite boundary mass in Euclidean space may by a 1960 theorem by Federer-Fleming be approximated in flat norm by polyhedral chains. For other coefficient groups or in other ambient spaces, one may, as in Fleming,1965 define flat chains as a flat distance completion of more elementary chains (such as polyhedral or Lipschitz chains). Throughout the years, flat chains have proven to be an important vehicle for Geometric Measure Theory applications to the study of variational, geometric, and topological properties of spaces. Briefly we will follow this development of flat chains through some results in works by Whitney 1957, Federer-Fleming 1960, Fleming 1966, Federer 1975, White 1999, Ambrosio-Kirchheim 2000, Wenger 2007, DePauw-Hardt 2010

Hopefully we can also say a few words about the parallel but sparser history of cochains.

4. Bruce Kleiner

- Title: Geometric measure theory and bilipschitz embeddings of metric spaces
- Abstract: Given metric spaces X and Y , one may ask if there is a bilipschitz embedding $X \rightarrow Y$, and if so, one may try to find an embedding with minimal distortion, or at least estimate the best bilipschitz constant. Such bilipschitz embedding problems arise in various areas of mathematics, including geometric group theory, Banach space geometry, and geometric analysis; in the last 10 years they have also attracted a lot of attention in theoretical computer science. A special case of particular interest in computer science is finding embeddings $X \rightarrow Y$ where the target space Y is the Banach space L^1 . This turns out to be closely tied to the structure of sets of finite perimeter in the domain X , and this leads to new applications of GMT.

The lecture will survey some of the background, and then focus on the role of GMT in embedding theory.

5. Leon Simon

- Title: The Allard Regularity Theorem
- Abstract: In the study of geometric and analytic problems which admit solutions with singularities, “ ϵ regularity theorems” typically play a key role. Such theorems apply to various classes of objects (typically special classes of submanifolds or functions) and, roughly speaking, assert that if an object lies in a suitable class, and if that object is “ ϵ -close” to a smooth member of the same class, then the object is itself smooth (i.e. has no singular points).

Such ϵ regularity theorems were first proved in the context of area minimizing hypersurfaces by De Giorgi, and later for more general classes of submanifolds by others, including Almgren and Allard.

The lectures will discuss the precise formulation and proof of Allard’s main ϵ regularity theorem, and will also give some indications about how the theorem applies to give information about the nature of the set of singular points.

6. Neshan Wickamasekera

- Title: Regularity of Stable Minimal Hypersurfaces

- Abstract: The focus of these lectures will be on the question of regularity of minimal hypersurfaces on a smooth Riemannian manifold. In its full generality—that is, assuming only that the hypersurface is a critical point of the area functional—very little is known concerning this question, and the lectures will start with a brief discussion of where the main difficulties lie. They will then focus on recent developments in the theory for stable minimal hypersurfaces, presenting what amounts to a complete theory giving embeddedness of the hypersurface away from a small singular set under a geometric structural condition. The lectures will conclude with a discussion of the situation in the absence of this structural condition.

7. Camillo DeLellis & Tatiana Toro

- Title: Geometric Measure Theory, the basics
- Abstract: The goal of this joint course is to present the basic notions in Geometric Measure Theory which constitute the required background for the other courses. The topics to be discussed include:
 - Radon measures, compactness and Riesz Representation Theorem
 - Hausdorff measures and densities
 - Lipschitz function and Radamacher's Theorem
 - Area and Co-area formulas
 - Rectifiable sets
 - Metric valued BV functions
 - First variation formula, generalized mean curvature and stationary rectifiable varifolds.