presenter: Steffen Rohde, University of Washington

title: Random metric spaces

problem: Define a random distance function $d_n(x,y)$ on the unit square $Q = [0,1]\times[0,1] \subset \mathbb{R}^2$ as follows. Denote $Q_{i_1 i_2 \cdots i_n}$ the dyadic squares of sidelength 2^{-n}, and let $w(Q_{i_1 i_2 \cdots i_n}) > 0$ be i.i.d. random variables. For each $I = i_1 \cdots i_n$, define the "size" of Q_I as

$$\sigma(Q_I) = \prod_{Q_J \supset Q_I} w(Q_J),$$

and the "length" of a curve $\gamma \subset Q$ as

$$\ell_n(\gamma) = \int_\gamma \sigma_n ds,$$

where $\sigma_n(x)$ is the size $\sigma(Q_I)$ of the generation-n square Q_I that contains x (if there are two such squares, just take the smaller size). Then define the distance

$$d_n(x,y) = \inf_\gamma \ell_n(\gamma),$$

where the infimum is over all curves in Q from x to y. The problem is to understand the metric space (Q, d_n) for large n. Specifically,

1) Show that the limit of $(Q, \frac{1}{E[\mathrm{diam}(Q,d_n)]} d_n)$ as $n \to \infty$ exists (as a distributional limit of random metric spaces, say with respect to Gromov-Hausdorff convergence).

Subsequential (distributional) limits always exist. So even without answering 1), it makes sense to ask:

2) Show that the subsequential limits are topological squares, a.s.

background:

Scaling limits of random triangulations of e.g. the sphere have been shown to exist by Le Gall, Miermont and others. The limits are conjectured to be related to Liouville Quantum Gravity. The above is a toy-model for Liouville Quantum Gravity. Another toy model is described by Scott Sheffield as problem 6 in *Conformal weldings of random surfaces: SLE and the quantum gravity zipper*, arXiv:1012.4797v1. If instead of the above metric $d_n = \sigma_n ds$ one considers the (random) measure $\mu_n = \sigma_n^2 dxdy$, then (weak) convergence of $\frac{1}{E[\mu_n(Q)]}\mu_n$ is known and goes under the name "multiplicative cascades". In dimension 1, there is a cheap way to extract a metric d from a measure μ, simply by setting $d(x,y) = \mu([x,y])$, but in higher dimensions such a simple device is not available.