Report on the
Tropical Structures in Geometry and Physics
Workshop
supported by
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Tropical structures in geometry and physics
Nov 29-Dec 4, 2009
workshop report
Organizers: M. Gross, K. Hori, R. Kenyon, V. Kharlamov

One of the successes of tropical geometry is in its applications to different areas of recently developing mathematics. Among these are enumerative geometry, symplectic field theory, mirror symmetry, dimer models/random surfaces, amoebas and algas, instantons, cluster varieties, and tropical compactifications.

We invited a diverse crowd of mathematicians and physicists whose work was united by the fact that it uses tropical geometric methods. We feel that the workshop was highly successful in allowing these different communities to interact and explain their research to each other. Because of these differences in background the speakers had to make a special effort—with varied success—to explain the motivations and terminology of their research. The MSRI setting: library, meeting rooms, and relative isolation lent itself very well to developing collaborations.

The following general themes summarize the main ideas at the conference:

1. Applications of tropical geometry to mirror symmetry.

2. Connections between tropical geometry and real geometry.

3. Connections with combinatorics.

4. Connections between tropical geometry and string theory

Tropical applications to mirror symmetry were reflected in talks of Zharkov, Markwig, Abouzaid, Hacking, Boehm and Parker. Tropical structures arise naturally in mirror symmetry, since near large complex structure limit points in complex moduli space, holomorphic structures are expected to converge to tropical structures.

Zharkov considered the mirror statement, exploring how tropical curves ((p,q)-webs in string theoretic terminology) are related to Lagrangian submanifolds on the mirror.

Markwig discussed her joint work with Johannes Rau on tropical gravitational descendent Gromov-Witten invariants. She demonstrated that one can give a purely synthetic computation of descendent invariants by defining psi classes as tropical cycles on the moduli space of tropical curves in projective two-space, and then carrying out tropical intersection theory to obtain descendent invariants. She then showed these coincide with genuine holomorphic descendent Gromov-Witten invariants. Markwig and Shustin lanched a collaboration on tropicalization of families of curves with deep singularities.

Abouzaid, in joint work Mark Gross and Bernd Siebert, discussed recent ideas of using tropical geometry to analyze part of the Fukaya category; again, one considers limiting

behaviour of holomorphic objects, in this case the holomorphic disks which arise in Floer theory.

Hacking, in joint work with Mark Gross and Sean Keel, applied tropical geometry to construct mirrors of rational surfaces with anti-canonical cycles of rational curves. As an application, he explained how this proved a 30 year old conjecture of Looijenga on the smoothability of cusp singularities.

Boehm discussed his work on constructing mirror pairs of Calabi-Yau manifolds using tropical geometry, coupled with Gröbner basis techniques. This work provides a generalization of the classical Batyrev-Borisov construction for complete intersections in toric varieties to other examples, such as Pfaffian Calabi-Yau manifolds.

Parker has developed a program for constructing relative Gromov-Witten invariants which is a vast generalization of the Li-Ruan construction of Gromov-Witten invariants relative a single divisor. This method uses tropical geometry to a certain extent, again in the context of a limiting picture for the constructions.

The talk by Passare attracted attention to various new aspects of the complex geometry of coamoebas that can be useful, for example, for developing a complexification of tropical geometry. It has stimulated joint works with J.-J.Risler and M.Nisse. An interesting idea for such a complexification was proposed by O.Viro. It attracted a wide interest and many discussions with many of the participants (Itenberg, Mikhalkin, Parker, Passare, and Zarkov, for example). After a talk by Parker, Viro and Kharlamov discussed with him relations between Parker's exploded manifolds and Viro's complex tropical geometry. They are actively continuing this collaboration.

Rares Rasdeaconu announced in his talk several new results obtained together with J. Solomon in the direction of constructing relative open Gromow-Witten invariants. Such invariants would open new ways in development of real enumerative geometry, for example. It was followed by discussions with Ionel, Itenberg, Kharlamov, Shustin.

As another interesting talk with fresh results was the talk by I.Zarkov who presented the current state of his collaboration with I.Itenberg, L.Katzarkov, and G.Mikhalkin on tropical Hodge theory.

Talks by Knutson, Speyer and Williams illustrated the uses of tropical methods in various combinatorics problems. Speyer and Kenyon discussed integrable structures arising in dimers and toric geometry; the connection with tropical geometry is not really clear yet. Williams and Kenyon began a discussion on the combinatorics of cluster algebras and dimer models.

Talks by Hanany, Krefl, Kol and Aganagic allowed to members of the program to become better aware of the approaches used in string theory by physicists in treating closely related objects and problems.

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Invited Speakers

lastname	firstname	institutionname
Abouzaid	Mohammed	MIT
Alessandrini	Daniele	Université de Strasbourg I (Louis Pasteur)
Boehm	Janko	Universitaet des Saarlandes, Campus E2 4
Brugalle	Erwan	Université Pierre et Marie Curie (Paris 6)
Hacking	Paul	University of Massachusetts
Hanany	Amihay	Imperial College London
Knutson	Allen	University of California, San Diego
Kol	Barak	Hebrew University
Krefl	Daniel	University of Tokyo
Litvinov	Grigory	Independent University of Moscow
Markwig	Hannah	Georg-August-Universität zu Göttingen
Parker	Brett	University of California
Passare	Mikael	University of Stockholm
Rasdeaconu	Rares	Hebrew University
Shustin	Eugenii	Tel Aviv University
Speyer	David	Massachusetts Institute of Technology
Tillmann	Stephan	University of Queensland
Tsikh	Avgust	Siberian Federal University
Williams	Lauren	University of California
Zharkov	Ilia	Kansas State University

MSRI Mathematical Sciences Research Institute

Tropical Structures in Geometry and Physics

November 30 to December 04

Monday November 30, 2009		
09:00AM - 10:00AM	Allen Knutson	Reduced degenerations and Frobenius splitting
10:00AM - 11:00AM	Coffee, tea in the atrium	
11:00AM - 12:00PM	Lauren Williams	Teichmuller space, cluster algebras from surfaces, and the positivity conjecture
12:00PM - 01:30PM	Lunch	
01:30PM - 02:30PM	David Speyer	Determinental hypersurfaces, convex polynomials and tropical geometry
02:30PM - 03:00PM	Coffee, tea in the atrium	
03:00PM - 04:00PM	Mikael Passare	Some aspects of discriminantal (co)amoebas
04:15PM - 05:15PM	Oleg Viro	TBD
05:15PM - 06:30PM	Reception in the atrium	
Tuesday December 1, 2009		
09:30AM - 10:30AM	Ilia Zharkov	Tropical (p,q)-classes of Lagrangian type
10:30AM - 11:00AM	Coffee, tea in the atrium	
11:00AM - 12:00PM	Eugenii Shustin	Real tropical enumerative invariants
12:00PM - 01:30PM	Lunch	
01:30PM - 02:30PM	Rares Rasdeaconu	Relative open Gromov-Witten invariants
02:30PM - 03:00PM	Coffee, tea in the atrium	
03:00PM - 04:00PM	Erwan Brugalle	Realizability of superabundant tropical curves
04:15PM - 05:15PM	Hannah Markwig	Tropical descendant Gromov-Witten invariants
Wednesday December 2, 2009		
09:30AM - 10:30AM	Daniele Alessandrini	On the compactification of the parameter space of convex projective structures
10:30AM - 11:00AM	Coffee, tea in the atrium	
11:00AM - 12:00PM	Mohammed Abouzaid	TBD
Thursday December 3, 2009		
09:30AM - 10:30AM	Daniel Krefl	Real enumerative geometry via the topological string
10:30AM - 11:00AM	Coffee, tea in the atrium	
11:00AM - 12:00PM	Amihay Hanany	(p,q) webs and their applications in string theory
12:00PM - 01:30PM	Lunch	
01:30PM - 02:30PM	Barak Kol	Tropical geometry and (p,q) webs
02:30PM - 03:00PM	Coffee, tea in the atrium	
03:00PM - 04:00PM	Mina Aganagic	Tropical Geometry and the Topological String
04:15PM - 05:15PM	Grigory Litvinov	Dequantization and tropical structures in classical mechanics and classical geometry
Friday December 4, 2009		
09:30AM - 10:30AM	Stephan Tillmann	The Hilbert geometry of the n-simplex
10:30AM - 11:00AM	Coffee, tea in the atrium	
11:00AM - 12:00PM	Paul Hacking	Smoothing surface singularities via mirror symmetry
12:00PM - 01:30PM	Lunch	
01:30PM - 02:30PM	Janko Boehm	Calabi-Yau mirrors via tropical geometry
02:30PM - 03:00PM	Coffee, tea in the atrium	
03:00PM - 04:00PM	Brett Parker	Tropical curves and Gromov Witten invariants

Currently Available Videos

- **Sam Payne** , Topology of compactified tropicalizations *October 12,2009, 09:30 AM to 10:30 AM*
- **Alex Esterov** , Newton polyhedra and Minkowski integrals. *October 12,2009, 11:00 AM to 12:00 PM*
- **Eric Katz** , Realization Spaces for Tropical Varieties *October 12,2009, 02:00 PM to 03:00 PM*
- **Arkady Berenstein** , Geometric crystals and tropical combinatorics *October 13,2009, 09:30 AM to 10:30 AM*
- **Gleb Koshevoy** , Bases of tropical Plucker functions, wirings, tilings and Leclerc-Zelevinsky conjectures. *October 13,2009, 11:00 AM to 12:00 PM*
- **Filip Cools** , Tropical geometry and dissimilarity vectors of trees *October 13,2009, 02:00 PM to 03:00 PM*
- **Daniele Alessandrini** , Tropicalization of Teichmuller spaces *October 13,2009, 04:00 PM to 05:00 PM*
- **Walter Gubler** , Tropical analytic geometry and the Bogomolov conjecture *October 14,2009, 09:00 AM to 10:00 AM*
- **Josephine Yu** , Linear Systems on Tropical Curves *October 14,2009, 10:30 AM to 11:30 AM*
- **Diane Maclagan** , Tropical bounds on effective cycles *October 14,2009, 11:45 AM to 12:45 PM*
- **Matthew Baker** , Metric properties of the tropical Abel-Jacobi map *October 15,2009, 09:30 AM to 10:30 AM*
- **Filippo Viviani** , On the tropical Torelli map *October 15,2009, 11:00 AM to 12:00 PM*
- **Sergey Fomin** , Enumeration of plane curves and labeled floor diagrams *October 15,2009, 02:00 PM to 03:00 PM*
- **Michael Joswig** , Coarse tropical convexity and cellular resolutions *October 16,2009, 09:30 AM to 10:30 AM*
- **Michael Joswig** , Brief Software Tutorial *October 16,2009, 10:30 AM to 11:00 AM*
- **Thorsten Theobald** , Combinatorics and genus of tropical intersections and Ehrhart theory *October 16,2009, 11:00 AM to 12:00 PM*
- **Alex Fink** , Tropical cycles and Chow polytopes *October 16,2009, 02:00 PM to 03:00 PM*
- **Annette Werner** , Buildings and tropical geometry *October 16,2009, 04:00 PM to 05:00 PM*You can find videos of other workshops and events on our VMath - Streaming Video page.

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Officially Registered Participants

lastname	firstname	institutionname
Abouzaid	Mohammed	MIT
Alessandrini	Daniele	Université de Strasbourg I (Louis Pasteur)
Amini	Omid	École Normale Supérieure
Auroux	Denis	University of California
Beil	Charles	University of California
Block	Florian	University of Michigan
Boehm	Janko	Universitaet des Saarlandes, Campus E2 4
Brugalle	Erwan	Université Pierre et Marie Curie (Paris 6)
Budreau	Dan	University of California, San Diego
Castano-Bernard	Ricardo	Kansas State University
Chan	Kwok Wai	Harvard University
Cueto	Maria Angelica	University of California
Devadoss	Satyan	Williams College
Diaz	Rafael	Universidad del Rosario
Dickenstein	Alicia	University of Buenos Aires
Diemer	Colin	University of Pennsylvania
Eager	Richard	University of California
Early	Nicholas	Louisiana State University
Gadbled	Agnès	Université de Neuchatel - Institut de Mathématiques
Garay	Cristhian	Institut Mathematiques de Jussieu
Gathmann	Andreas	University of Kaiserslautern
Gross	Mark	University of California, San Diego
Haas	Ruth	Smith College
Hacking	Paul	University of Massachusetts
Hanany	Amihay	Imperial College London
Haque	Mohammad	University of Texas
Helminck	Aloysius	North Carolina State University
Herold	Matthias	TU Kaiserslautern
Hori	Kentaro	University of Toronto
Hower	Valerie	University of California
Ibrahim	Ashraf	Texas A & M University
Itenberg	Ilia	Institut de Recherche Mathématique Avancée de Strasbourg
Izhakian	Zur	Bar-Ilan University
Jensen	Anders	Georg-August-Universität zu Göttingen
Johansson	Petter	Stockholm University
Karaali	Gizem	Pomona College
Kenyon	Richard	Brown University
Knutson	Allen	University of California, San Diego
Kol	Barak	Hebrew University
Krefl	Daniel	University of Tokyo
Leung	Naichung	Chinese University of Hong Kong
Litvinov	Grigory	Independent University of Moscow
Lundqvist	Johannes	University of Stockholm
Madani	Farid	Université de Paris VI (Pierre et Marie Curie)
Markwig	Hannah	Georg-August-Universität zu Göttingen

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Matessi	Diego	Università del Piemonte Orientale ``Amedeo Avogadro''
McCrory	Clinton	University of Georgia
Meyer	Henning	Universität Kaiserslautern
Munguia	Erendira	National Autonomous University of Mexico (UNAM)
Musiker	Gregg	MIT, Department of Mathematics
Nairn	Kristen	St. John's University
Nesci	Michele	Université de Genève
Novoseltsev	Andrey	University of Alberta
Ochse	Dennis	Universität Kaiserslautern
Parker	Brett	University of California
Pascaleff	James	Massachusetts Institute of Technology
Passare	Mikael	University of Stockholm
Rasdeaconu	Rares	Hebrew University
Rowen	Louis	Bar-Ilan University
Schroeter	Franziska	Georg-August-Universität zu Göttingen
Severs	Christopher	Arizona State University
Shaw	Kristin	University of Toronto
Sheridan	Nicholas	Massachusetts Institute of Technology
Shustin	Eugenii	Tel Aviv University
Skoldberg	Emil	National University of Ireland, Galway
Slawinski	Mike	University of California
Speyer	David	Massachusetts Institute of Technology
Sverdlov	Roman	Raman Research Institute
Taliwal	Vikas	Cornell University
Thieullen	Philippe	Université de Bordeaux I
Tillmann	Stephan	University of Queensland
Torchiani	Carolin	Technical University of Kaiserslautern
Tsikh	Avgust	Siberian Federal University
Tyomkin	Ilya	Department of Mathematics, Ben Gurion University of the Negev
Venkatram	Kartik	Boston University
Werner	Annette	Johann Wolfgang Goethe-Universität Frankfurt
Williams	Lauren	University of California
Yu	Josephine	Massachusetts Institute of Technology
Zabun	Arzu	Middle East Technical University (ODTU)
Zharkov	Ilia	Kansas State University

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Officially Registered Participant Information

Participants		**80**

Gender		**80**
Male	76.25%	61
Female	20.00%	16
Declined to state	3.75%	3

Ethnicity*		**82**
White	69.51%	57
Asian	9.76%	8
Hispanic	7.32%	6
Pacific Islander	0.00%	0
Black	1.22%	1
Native American	0.00%	0
Declined to state	12.20%	10

* ethnicity specifications are not exclusive

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NSA Financial Report

	NSF	NSA	Private	Total
Per diem	$ 14,988.00	$ 10,280.00		$ 25,268.00
Travel	$ 4,599.67	$ 4,720.00		$ 9,319.67
Food	$ 2,022.49			$ 2,022.49
Notetaker	$ 250.00			$ 250.00
Total	**$ 21,860.16**	**$ 15,000.00**	**$ -**	**$ 36,860.16**

NSA Per Diem	
Offered	$ 10,280.00
Actual	$ 10,280.00

NSA Travel	
Offered	$ 4,720.00
Actual	$ 4,720.00

	US Cit or PR	Per Diem	Travel	Total
Beil, Charles	US Citizen	$ 560.00	$ 181.03	$ 741.03
Budreau, Daniel	US Citizen	$ 700.00	$ 250.00	$ 950.00
Diemer, Colin	US Citizen	$ 700.00		$ 700.00
Eager, Richard	US Citizen	$ 700.00	$ 250.00	$ 950.00
Falk, Michael	US Citizen	$ 700.00		$ 700.00
Hacking, Paul Anthony	US PR	$ 660.00		$ 660.00
Haque, Mohammad Moinul	US Citizen	$ 700.00	$ 537.41	$ 1,237.41
Katzarkov, Ludmil	US Citizen	$ 700.00		$ 700.00
Kenyon, Richard	US PR	$ 700.00	$ 550.00	$ 1,250.00
Knutson, Allen	US PR	$ 660.00	$ 550.00	$ 1,210.00
Kol, Barak	US Citizen	$ -	$ 621.76	$ 621.76
Nairn, Kristen Ann	US Citizen	$ 700.00	$ 550.00	$ 1,250.00
Rowen, Louis	US Citizen	$ 700.00	$ 492.00	$ 1,192.00
Slawinski, Mike	US Citizen	$ 700.00	$ 221.20	$ 921.20
Sverdlov, Roman Markovich	US Citizen	$ 700.00		$ 700.00
Venkatram, Kartik	US Citizen	$ 700.00	$ 516.60	$ 1,216.60
Grand Total		**$ 10,280.00**	**$ 4,720.00**	**$ 15,000.00**