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title: Rough isometry between random sets of integers

problem:

Let X and Y be metric spaces; a map $\phi : X \rightarrow Y$ is said to be a *rough isometry* if there are constants $M, D, R > 0$ such that

1. for every $u, v \in X$, $\frac{1}{M}d_X(u, v) - D \leq d_Y(\phi(u), \phi(v)) \leq Md_X(u, v) + D$ and
2. for every $y \in Y$, there is an $x \in X$ such that $d_Y(\phi(x), y) \leq R$.

Let \mathbb{Z}_p be the result of Bernoulli percolation on the integers \mathbb{Z} , that is, each k is put in \mathbb{Z}_p independently with probability p . The metric is inherited from \mathbb{Z} , so that $d(j, k) = |j - k|$.

Question: are there $p, q \in (0, 1)$ such that with positive probability, \mathbb{Z}_p and \mathbb{Z}_q are roughly isometric? Equivalently, is it the case that for *every* $p, q \in (0, 1)$, \mathbb{Z}_p and \mathbb{Z}_q are roughly isometric with probability 1? (Also equivalently, the same questions about Poisson processes on \mathbb{R} .)

background:

The concept of rough isometry was introduced in group theory by Gromov in the late '70s, and studied in a wider context by Kanai in '85. Miklós Abért asked whether the infinite clusters of two percolations on the Cayley graph of a finitely-generated group are roughly isometric; Balász Szegedy then asked more specifically about site percolation on (all of) \mathbb{Z}^2 , then \mathbb{Z} . For more information and partial results, see Ron Peled's article in *Ann. Appl. Prob* **10** #2 (2010), pp. 462–494, <http://arxiv.org/abs/0709.2383>.