

# 1 SCIENTIFIC GOALS

In the Spring of 2008 two half-year programs took place at MSRI: *Combinatorial Representation Theory* and *Representation Theory of Finite Groups*. What the two programs had in common was the central role played by Lie theory. In combinatorial representation theory the most important combinatorial objects used to model representations arise from Lie theory (tableaux, Littlewood-Richardson coefficients, Kazhdan-Lusztig polynomials, etc.). In finite groups there are several trends making Lie theory central. Some finite groups are either naturally a part of Lie theory (finite groups of Lie type) or are very closely connected to it on many levels (symmetric groups). Central general conjectures in representation theory of finite groups can often be tested on these important classes of groups and sometimes, using the Classification Theorem, might even be reduced to questions about these groups. On the other hand, the theory of  $p$ -compact groups provides a bridge between a finite group theory, algebraic topology and complex reflection groups. So it was very natural for the two MSRI programs to run a joint workshop on *Lie Theory*.

# 2 ORGANIZATIONAL STRUCTURE

The workshop has been organized following a standard MSRI 5 day long research workshop template. There were four 60 minute talks each day, except for Wednesday, when there were just two 60 minute talks in the morning. The structure seems to be optimal, as the uncrowded talk schedule left enough time for discussion. Each speaker has been given a short introduction by a ‘classic’ in that area (for example Robert Steinberg presented George Lusztig, Charley Curtis presented Meinolf Geck, etc.).

The new facilities at MSRI were positively commented on by many participants. Seating, visibility, blackboards, multimedia, etc. all seem outstanding.

# 3 SCIENTIFIC DEVELOPMENTS

Let us mention in the beginning that a general level of excitement of all participants was very high. Many participants and some UC-Berkeley faculty members gave very favorable reviews to the workshop, which has been one of the biggest such events in the history of MSRI. Significant research accomplishments were described in many talks. The auditorium was packed during all talks of the workshop.

We now describe some of the most significant talks of the conference. Complete list of talks is given in an appendix below.

**Victor Ostrik (University of Oregon)** presented a major development on Lusztig’s asymptotic Hecke ring (joint work with Bezrukavnikov and Finkelberg). As explained by Lusztig, the asymptotic Hecke ring plays a significant role in represen-

tation theory of finite groups of Lie type and in the theory of character sheaves. The explicit description of the asymptotic Hecke ring for each two-sided cell has been conjectured by Lusztig. The authors prove the Lusztig's conjecture in a very satisfactory way, using the theory of tensor categories. It turns out that the conjecture holds true for all non-exceptional cells, of which there is only one in type  $E_7$  and two in type  $E_8$ . What is remarkable, the classification results on monoidal tensor categories provide an explanation of what goes wrong in the exceptional cases, and provide a complete solution in these cases also. In the end of the talk, Ostrik presented an exciting and important new conjecture, due to him and Bezrukavnikov, which connects Lusztig's asymptotic ring to representation theory of finite  $W$ -algebras. This has provided a link to the number of talks on the conference which were dedicated to finite  $W$ -algebras, and generated a lot of discussion between the experts (Ostrik, Premet, Losev, Kleshchev, Goodwin, Brown).

**Alexander Premet (University of Manchester)** and **Ivan Losev (Belarusian State University)** presented two significant talks on representation theory of *finite  $W$ -algebras*. One of the major problems of the theory is classification of finite dimensional representations of these algebras. This is closely related to the theory of primitive ideals in universal enveloping algebras, deformations of singularities, quiver representations, and physics, to name just a few connections. On the other hand, very little is known about representation theory of finite  $W$ -algebras outside of type  $A$  (which has been treated by Brundan and Kleshchev). Until recent work of Premet it has not even been known if in general finite  $W$  algebras have *any* finite dimensional representations. Even more importantly is a result conjectured by Premet that 1-dimensional representation always exists. In his talk Premet presented a very interesting solution of this problem for classical types using reduction modulo  $p$  and lifting back, as well as results of Barbash and Vogan on primitive ideals. On the other hand, Ivan Losev presented a completely different approach to  $W$ -algebras based on the ideas of *Fedosov quantization*. He observe that the finite  $W$ -algebras  $W$  is the invariant algebra for an action of a reductive group  $G$  with Lie algebra  $\mathfrak{g}$  on a quantized symplectic affine variety and used this observation to study  $W$ . The results include an alternative definition of  $W$ , a relation between the sets of prime ideals of  $W$  and of the corresponding universal enveloping algebra, the existence of a one-dimensional representation of  $W$  in the case of classical Lie algebra, and the separation of elements of  $W$  by finite dimensional representations.

**Meinolf Geck (University of Aberdeen)** gave a spectacular talk on James conjecture for Hecke algebras of exceptional type. Originally, James conjecture is concerned with modular representation theory of symmetric groups. Roughly speaking it describes characters of irreducible representations under certain non-obvious restrictions in terms of the corresponding theory for Hecke algebras at roots of unity. It can also be described as a (non-obvious) symmetric group analogue of Lusztig's conjecture for algebraic groups. It needs to be said that the conjecture is completely

open, and, moreover, it is far from clear what the analogue for other types should look like. Recently Geck showed that Hecke algebras of finite type are cellular in the sense of Graham-Lehrer. This led to a natural generalisation of the theory of Specht modules to Hecke algebras of any (finite) type. In this framework, he also formulated a general version of James' conjecture. In a joint work with Juergen Mueller, he now proved James' conjecture for Hecke algebras of exceptional type.

**George Lusztig (MIT)** made a foundational talk on reductive algebraic groups. He explained how to use canonical bases theory to prove the result announced but left unproved by Kostant in 1966! The result is an explicit construction of the coordinate ring of a reductive algebraic group over the integers. Lusztig explained why such a proof could not be found in 1966.

**Sergei Fomin (University of Michigan)** gave an expository lecture on cluster algebras. This talk was part of the series of MSRI *Evans talks*—lecture series designed especially for students and faculty members of UC-Berkeley and given at the Evans Hall on campus. Cluster algebras arise in various algebraic and geometric contexts, with combinatorics providing a unifying framework. The presentation of the basic definitions and results of this emerging theory was guided by two sets of examples: coordinate rings of classical algebraic varieties, and cluster algebras associated with bordered oriented surfaces with marked points. The topic of *cluster algebras* has been continued in the talk of **Bernard Leclerc (University of Caen)** who introduced the notion of monoidal categorification of a cluster algebra, and gave examples of such categorifications coming from the representation theory of quantum affine algebras.

**Peter Fiebig (University of Freiburg)** gave a talk on Lusztig's conjecture on characters of irreducible representation of algebraic groups over an in positive characteristic. Fiebig related sheaves of vector spaces on a complex affine flag variety to representations of the Lie algebra associated to Langlands dual root system. From this he extracted a new proof of Lusztig's multiplicity conjecture for almost all characteristics. The main step in the construction of the above relation was a categorification of a natural map from the affine Hecke algebra to its periodic module via the theory of sheaves on moment graphs. Using this categorification a non-topological proof of the multiplicity one case of Lusztig's conjecture for all characteristics above the Coxeter number was presented.

**Edward Frenkel (UC-Berkeley)** discussed a conjectural description of the categories assigned by the local geometric Langlands correspondence assigns to a local system on the punctured disc for the Langlands dual group of a complex reductive group  $G$ . The categories are given as categories of representations of the corresponding affine Kac-Moody algebra of critical level. Sometimes these categories may also be realized as categories of D-modules or O-modules on some algebraic varieties or ind-schemes. Interrelations between these categories provide supporting evidence for the conjectures. In particular, the categories of Iwahori equivariant representations of critical level with fixed central character are equivalent to the categories of quasi-

coherent sheaves on the Springer fibers the Langlands dual group.



# *Lie Theory*

March 10, 2008 to March 14, 2008

## Schedule

### Monday March 10, 2008

9:30AM - 10:30AM	Georgia Benkart	Bases, Lattices, Hyperbolic Kac-Moody Algebras
11:00AM - 12:00PM	Victor Ginzburg	Mirabolic affine Grassmannian and character sheaves
12:00PM - 2:00PM	Lunch	
2:00PM - 3:00PM	Viktor Ostrik	Tensor categories attached to cells in Weyl groups.
3:00PM - 4:00PM	Tea	
4:10PM - 5:00PM	Sergey Fomin	Evans talk (60 Evans Hall): Cluster algebras

### Tuesday March 11, 2008

9:30AM - 10:30AM	George Lusztig	Canonical bases and construction of Chevalley groups over $\mathbb{Z}$
11:00AM - 2:00PM	Peter Littelmann	Superbosonization of invariant matrix ensembles
12:00PM - 2:00PM	Lunch	
2:00PM - 3:00PM	Bernard Leclerc	Monoidal categorifications of cluster algebras
3:00PM - 3:30PM	Tea	
3:30PM - 4:30PM	Peter Fiebig	Sheaves on affine flag varieties, modular representations and Lusztig's conjecture

## Wednesday March 12, 2008

9:30AM - 10:30AM	Alexander Premet	Small nonrestricted representations and completely prime primitive ideals.
11:00AM - 2:00PM	Cedric Bonnafé	Semicontinuity properties of Kazhdan-Lusztig cells

## Thursday March 13, 2008

9:30AM - 10:30AM	Ivan Loseu	Quantized symplectic actions and W-algebras
11:00AM - 12:00PM	Edward Frenkel	Langlands correspondence for loop groups
12:00PM - 2:00PM	Lunch	
2:00PM - 3:00PM	Vyjayanthi Chari	Abelian Ideals, KR--modules and Koszul algebras
3:00PM - 4:00PM	Tea	
4:00PM - 5:00PM	Victor Kac	Poisson vertex algebras and integrable systems (!at EVANS HALL 60!)

## Friday March 14, 2008

9:30AM - 10:30AM	Meinolf Geck	James' conjecture for Hecke algebras of exceptional type
11:00AM - 2:00PM	Susumu Ariki	Two applications of Littelmann's path model
12:00PM - 1:30PM	Lunch	
1:30PM - 2:30PM	Gustav Lehrer	Coxeter group actions on the cohomology of toric varieties
2:30PM - 3:00PM	Tea	
3:00PM - 4:00PM	Gunter Malle	TBD

## Currently Available Videos

- Georgia Benkart , Bases, Lattices, Hyperbolic Kac-Moody Algebras March 10,2008, 09:30 AM to 10:30 AM
- Victor Ginzburg , Mirabolic affine Grassmannian and character sheaves March 10,2008, 11:00 AM to 12:00 PM
- Viktor Ostrik , Tensor categories attached to cells in Weyl groups March 10,2008, 02:00 PM to 03:00 PM
- Sergey Fomin , Cluster algebras March 10,2008, 04:10 PM to 05:00 PM
- George Lusztig , Canonical bases and construction of Chevalley groups over  $\mathbb{Z}$  March 11,2008, 09:30 AM to 10:30 AM
- Peter Littelmann , Superbosonization of invariant matrix ensembles March 11,2008, 11:00 AM to 12:00 PM
- Bernard Leclerc , Monoidal categorifications of cluster algebras March 11,2008, 02:00 PM to 03:00 PM
- Peter Fiebig , Sheaves on affine flag varieties, modular representations and Lusztig's conjecture March 11,2008, 03:30 PM to 04:30 PM
- Alexander Premet , Small nonrestricted representations and completely prime primitive ideals. March 12,2008, 09:30 AM to 10:30 AM
- Cedric Bonnafé , Semicontinuity properties of Kazhdan-Lusztig cells March 12,2008, 11:00 AM to 12:00 PM
- Ivan Loseu , Quantized symplectic actions and  $W$ -algebras March 13,2008, 09:30 AM to 10:30 AM
- Edward Frenkel , Langlands correspondence for loop groups March 13,2008, 11:00 AM to 12:00 PM
- Vyjayanthi Chari , Abelian Ideals,  $KR$ --modules and Koszul algebras March 13,2008, 02:00 PM to 03:00 PM
- Victor Kac , Poisson vertex algebras and integrable systems March 13,2008, 04:00 PM to 05:00 PM
- Meinolf Geck , James' conjecture for Hecke algebras of exceptional type March 14,2008, 09:30 AM to 10:30 AM
- Susumu Ariki , Two applications of Littelmann's path model March 14,2008, 11:00 AM to 12:00 PM
- Gustav Lehrer , Coxeter group actions on the cohomology of toric varieties March 14,2008, 01:30 PM to 02:30 PM
- Gunter Malle , Counting Characters of  $p'$ -degree March 14,2008, 03:00 PM to 04:00 PM

Name	Role	Institution
Alghamdi, Ahmad M.	Participant	Umm Alqura University
Andersen, Henning Haahr	Participant	Aarhus Universitet
Andikfar, Hossein	Participant	University of Toledo
Andre, Carlos	Participant	University of Lisbon
Andruskiewitsch, Nicolas	Participant	Universidad Nacional de Cordoba
Ariki, Susumu	Speaker	Kyoto University
Armstrong, Drew	Participant	University of Minnesota
Assaf, Sami H	Participant	MIT
Banu, Letitia Mihaela	Participant	University of Western Ontario
Barcelo, H�el�ene	Participant	MSRI
Barnet-Lamb, Thomas James	Participant	Harvard University
Batra, Punita	Participant	Harish Chandra Research Institute
Beck, Matthias	Participant	San Francisco State University
Benkart, Georgia M.	Speaker	University of Wisconsin
Benson, Dave J.	Participant	University of Aberdeen
Bessenrodt, Christine	Participant	Leibniz Universitaet Hannover
bidkhor, hoda	Participant	Massachusetts Institute of Technology
Boltje, Robert	Participant	University of California, Santa Cruz
Bonnafe, Cedric	Speaker	Centre National de la Recherche Scientifique
Bouc, Serge	Participant	CNRS - Universit� de Picardie
Brou�, Michel	Speaker	Institut Henri Poincar�
Brown, Jonathan	Participant	University of Oregon
Bunke, Thomas	Participant	Universidade de Sao Paulo
Bystron, Jakub	Participant	Charles University, Prague
Campbell, Peter	Participant	University of Bristol
Can, Mahir Bilen	Participant	university of western ontario
Carlson, Jon F.	Participant	University of Georgia
Carrell, James B.	Participant	University of British Columbia
Chari, Vyjayanthi	Speaker	University of California, Riverside
Chebolu, Sunil Kumar	Participant	University of Western Ontario
Cheng, Shun-Jen	Participant	Academia Sinica
Cherniavsky, Yonah	Participant	Technion -- Israel Institute of Technology
coleman, paula	Participant	cima pharmaceuticals
Cramer, Tim	Participant	Yale University
Craven, David A	Participant	University of Oxford
Curtis, Charles W.	Participant	University of Oregon
Danz, Susanne	Participant	University of Jena
Daugherty, Zajj	Participant	University of Wisconsin
Davis, Matt	Participant	University of Wisconsin
Denton, Tom	Participant	University of California
Dobria, Eunice Voichita	Participant	Florida Atlantic University
Douglass, J. Matthew	Participant	University of North Texas
Dudas, Olivier	Participant	Laboratoire de Mathematiques de Besan�on
Edwards, Robert	Participant	UCLA
Ehrig, Michael	Participant	University of Cologne
Elliot, Jason Walter	Participant	University of Illinois
Fayers, Matthew	Participant	Massachusetts Institute of Technology
Feldvoss, Joerg	Participant	University of South Alabama
Fiebig, Peter	Participant	Universit�t Freiberg
Fishel, Susanna Dodds	Participant	Arizona State University
Fomin, Sergey	Speaker	University of Michigan
Fong, Paul	Participant	University of Illinois, Chicago
Fourier, Ghislain	Participant	Universit�t zu K�ln
Frenkel, Edward	Speaker	UCB - University of California, Berkeley
Futory, Vyacheslav	Participant	Universidade de Sao Paulo
Gan, Wee Teck	Participant	Princeton University
Gaussent, Stephane	Participant	Universite de Nancy



Geck, Meinolf J	Speaker	King's College, Aberdeen University
Geiss, Christof	Participant	Universidad Nacional Autonoma de Mexico
Ghanam, Ryad	Participant	University of Pittsburgh-Greensburg
Ginzburg, Victor A.	Speaker	University of Chicago
Gonzales, Richard	Participant	NPR
Goodman, Frederick	Participant	University of Iowa
Goodwin, Simon	Participant	University of Birmingham
Gorelik, Maria	Participant	Weizmann Institute of Science
Graber, John	Participant	University of Iowa
Greene, Curtis	Participant	Haverford College
Guilhot, Jeremie	Participant	University of Aberdeen
Guralnick, Robert M.	Participant	USC
Haglund, James B.	Participant	Univ. of Pennsylvania
Haiman, Mark David	Participant	UCB - University of California, Berkeley
He, Xuhua	Participant	Stony Brook University
Helminck, Aloysius Gerardus	Participant	NC State University
Hemmer, David J.	Participant	State University at Buffalo, SUNY
Hill, David Edward	Participant	University of California, Berkeley
Himstedt, Frank	Participant	Technische Universität Munchen
Hiss, Gerhard Richard	Participant	RWTH Aachen
Hoshino, Ayumu	Participant	University of Tokyo
Hoyt, Crystal Faye	Participant	Weizmann Institute of Science
Ip, Ivan	Participant	Yale University
jacon, Nicolas	Participant	Universite de Franche comte
Jakelic, Dijana	Organizer	University of Illinois at Chicago
Johnson, Garrett	Participant	University of California at Santa Barbara
Jones, Benjamin F.	Participant	University of Georgia
Jones, Brant	Participant	University of California, Davis
Juan, Lourdes	Participant	Texas Tech University
Juteau, Daniel Pierre	Participant	Jussieu University
Kac, Victor	Speaker	MIT - Massachusetts Institute of Technology
Kasatani, Masahiro	Participant	Kyoto University
Kashiwara, Masaki	Speaker	L'Institut de Mathématiques de Jussieu
Kashuba, Iryna	Participant	University of Sao Paulo
Kedem, Rinat	Participant	University of Illinois, Urbana-Champaign
Khare, Apoorva	Participant	University of California at Riverside
Khongsap, Totrakool	Participant	University of Virginia
KIM, SungSoon Yj	Participant	University of Paris 7
Kivran-Swaine, Terence Joseph	Participant	The Graduate Center of CUNY
Kleshchev, Alexander	Organizer	University of Oregon
Konvalinka, Matjaz	Participant	MIT
Krause, Henning	Participant	University of Paderborn
Kuelshammer, Burkhard	Participant	University of Jena
Kujawa, Jonathan R	Participant	University of Oklahoma
Kuwabara, Toshiro	Participant	Kyoto University
Kwon, Namhee	Participant	University of Toledo
Lam, Thomas	Participant	Harvard University
Lasy, Trafim	Participant	Paris 7
Lau, Michael	Participant	University of Windsor
Lauda, Aaron D	Participant	Columbia University
Lauve, Aaron	Participant	Texas A&M University
Le, Tung Thien	Participant	Wayne State University
Leclerc, Bernard	Speaker	Université de Caen
Lehrer, Gustav I.	Speaker	University of Sydney
Li, Yiqiang	Participant	Yale University
Lien, TseChing	Participant	University of Wisconsin, Madison
Lin, Zongzhu	Participant	Kansas State University
Lu, Dan	Participant	Yale University

Lusztig, George	Speaker	MIT
Lyle, Sinead	Participant	University of East Anglia
Lynd, Justin	Participant	The Ohio State University
Malle, Gunter	Participant	TU Kaiserslautern
Marko, Frantisek	Participant	PennState University Hazleton
Maroti, Attila	Participant	University of Southern California
Mazza, Nadia	Participant	University of Aberdeen
Mbirika, Abukuse (Aba)	Participant	University of Iowa
Milas, Antun	Participant	Rutgers University, New Brunswick
Miyachi, Hyohe	Participant	Nagoya University
Moci, Luca	Participant	Roma Tre
Moreau, Anne	Participant	ETH Zürich
Morier-Genoud, Sophie	Participant	University of Michigan
Musiker, Gregg	Participant	Massachusetts Institute of Technology
Nakanishi, Tomoki	Participant	Nagoya University
Nakano, Daniel	Participant	University of Georgia
Nakashima, Toshiki	Participant	Sophia University
Nash, David	Participant	University of Oregon
Nevins, Monica	Participant	University of Ottawa
Nguyen, Hung Ngoc	Participant	University of Florida
Noeske, Felix	Participant	RWTH Aachen University
O'Brien, Eamonn A	Participant	University of Auckland
Ohn, Christian	Participant	Universite de Valenciennes
Okada, Soichi	Participant	Nagoya University
Orellana, Rosa	Participant	Dartmouth College
Ostrik, Viktor	Speaker	University of Oregon
Pal, Tanusree	Participant	Harish-Chandra Research Institute
Park, Sejong	Participant	University of Aberdeen
Premet, Alexander	Speaker	University of Manchester
Purbhoo, Kevin	Participant	University of Waterloo
Ragnarsson, Kari	Participant	University of Illinois, Chicago
Rainbolt, Julianne Geering	Participant	St. Louis University
Ram, Arun	Organizer	University of Melbourne
Reif, Shifra	Participant	Weizmann institute of Science
Roehrl, Gerhard Erich	Participant	University of Bochum
Roichman, Yuval	Participant	Bar-Ilan University
Roth, Ilan	Participant	UC Berkeley
SanAgustin, Keefe L	Participant	Brandeis University
Saxl, Jan	Participant	University of Cambridge
Schilling, Anne	Participant	University of California, Davis
Scott, Leonard	Participant	University of Virginia
Serrano, Luis Guillermo	Participant	University of Michigan
Shchigolev, Vladimir	Participant	Lomonosov Moscow State University
Shinkado, Takuya	Participant	Kyoto university
Shtukaturov, Konstantin Yurievich	Participant	Institute of Power Transportation
Skoda, Zoran	Participant	Institute Ruder Boskovic
Smirnov, Evgeny	Participant	Universitaet Bonn
Solomon, Louis	Participant	University of Wisconsin
Srinivasan, Bhama	Participant	University of Illinois, Chicago
Stanley, Richard P.	Organizer	MIT - Massachusetts Institute of Technology
Steinberg, Robert	Participant	UCLA
Stokke, Anna	Participant	University of Winnipeg
Stump, Christian	Participant	University of Vienna
Sun, Jie	Participant	University of Alberta
Swenson, Daniel	Participant	University of Minnesota
Symonds, Peter	Participant	University of Manchester
Tan, Kai Meng	Participant	National University of Singapore
Taskin, Muge	Participant	York University

Thiem, Nat	Participant	University of Colorado
Tiep, Pham Huu	Participant	University of Florida
Tikaradze, Akaki	Participant	The University of Chicago
Tingley, Peter	Participant	UC Berkeley
Townsley, Lisa	Participant	Benedictine University
Tsuchioka, Shunsuke	Participant	Kyoto University
Vazirani, Monica Joy	Participant	University of California, Davis
Virk, Rahbar	Notetaker	University of Wisconsin
Walia, Rajeev	Participant	University of California Riverside
Walter, Marty	Participant	University of Colorado
Wan, Jinkui	Participant	University of Virginia
Wang, Weiqiang	Participant	University of Virginia
Watanabe, Hidekazu	Participant	University of Tokyo
Watson, Holly Lynn	Participant	University of South Carolina
Webb, Peter	Participant	University of Minnesota
Wiesner, Emilie	Participant	Ithaca College
Williams, Lauren Kiyomi	Participant	Harvard University
Williamson, Geordie	Participant	Universität Freiburg
Witherspoon, Sarah	Participant	Texas A&M University
Yamamoto, Makoto	Participant	University of California
Yip, Martha	Notetaker	University of Wisconsin
Yoo, Meesue	Participant	University of Pennsylvania
Zhu, Minxian	Participant	Yale University
Zorrilla Masías, Henry	Participant	Universidad Nacional Federico Villarreal