

**presenter:** Svante Linusson, KTH, Sweden

**title:** The Bunkbed conjecture

**problem:** Let  $G$  be any finite graph and call  $\tilde{G} := G \times K_2$  the corresponding *bunkbed graph*. Here  $K_2$  consists of two vertices  $\{0, 1\}$  connected by an edge. Think of the bunkbed graph as one lower copy of  $G$  and one upper copy of  $G$  and vertical edges between  $u_0$  and  $u_1$  for every vertex  $u$  in  $G$ . Now, consider bond percolation on  $\tilde{G}$  with probability  $p$ , i.e. every edge in  $\tilde{G}$  exists with probability  $p$  independently of all other edges.

**Conjecture 0.1** (Kasteleyn '85). *For every graph  $G$ , every pair of vertices  $s, e$  in  $G$  and every  $0 \leq p \leq 1$  we have*

$$\mathbb{P}(s_0 \leftrightarrow e_0) \geq \mathbb{P}(s_0 \leftrightarrow e_1),$$

*i.e. the probability that there is a path from  $s_0$  to  $e_0$  is greater than that there is a path from  $s_0$  to  $e_1$ .*

The conjecture has been proven for  $G$  not having  $K_{2,3}$  as a minor, see [L], and also if one instead of bond percolation consider the random cluster model with  $q = 2$ , see [H].

**Background** To the best of my knowledge this conjecture was first posed by Kasteleyn. His formulation seems to have been slightly more general, in that he conditioned on the set of vertical edges being present and conjectured it to be true for any such set  $T$ , see [vBK]. It is easy to see that it is true if  $|T| \leq 1$ , but already for  $|T| = 2$ , I do not know of a proof for any graph.

[vBK] Jacob van den Berg and Jeff Kahn, 'A correlation inequality for connection events in percolation', *Annals of Probability* **29** No. 1 (2001), 123–126.

[H] Olle Häggström, Probability on Bunkbed Graphs, *Proceedings of FPSAC'03, Formal Power Series and Algebraic Combinatorics* Linköping, Sweden 2003. Available online at

<http://www.fpsac.org/FPSAC03/ARTICLES/42.pdf>

[L] Svante Linusson, On percolation and the bunkbed conjecture, *Combinatorics, Probability and Computing* **20** No. 1 (2011), 103-117.