presenter: Rick Kenyon, Brown University

title: Homologically Trivial Walks

problem:

Given an infinite graph G, for example $\mathbb{Z}^2$, and vertex v, let $f_v(n)$ be the number of simple walks of length n, starting at v, which cross each edge of G zero times algebraically, that is, cross each edge an equal number of times in each direction. Evaluate $f_v(n)$.

background: When $G = \mathbb{Z}^2$, simple walks without backtracks are in bijection with elements of the free group $F(a, b)$, where a and a^{-1} represent horizontal steps right or left, and b, b^{-1} vertical steps up or down. With the added proviso that we want backtrack-free walks, then the number $f_e(n)$ of homologically trivial walks is the growth of the second commutator subgroup $[[F, F], [F, F]]$.

When G is finite, one can argue that the $f_v(n) \sim C\mu^n n^{-d/2}$ where d is the dimension of the cycle space of G, and μ is the growth rate of all simple walks. The idea is to consider the maximal abelian cover $\tilde{G}$ (which has a $\mathbb{Z}^d$-action with quotient G). Homologically trivial walks on G are in bijection with walks on $\tilde{G}$ which go from a fixed lift $\tilde{v}$ of v back to itself (i.e. loops).