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title: Random Eulerian Circuits

problem: A finite connected balanced directed graph has a Eulerian circuit, so on such a graph one can consider a uniform random Eulerian circuit $\mathcal{C}$. There is a simple algorithm – see e.g. Kandel - Matias - Unger - Winkler *Shuffling biological sequences* – for simulating $\mathcal{C}$. Use the random walk method for simulating a uniform spanning tree, and regard that tree as directed toward an arbitrary root. This specifies the final exit the circuit will make from each non-root vertex. Now start the circuit at the root, and at each step choose uniformly from the possible edges not previously used, saving the pre-specified exit for last.

But it is not easy to extract properties of $\mathcal{C}$ from this construction, and there is essentially no literature on properties of $\mathcal{C}$. Here is one of many special cases one might study.

Take the d-dimensional discrete torus Z_N^d and change each edge to two directed edges, giving $2dN^d$ dirccted edges. A uniform random Eulerian circuit will have $2d$ excursions from the origin: write the lengths of these excursions in increasing order as $L_1 \le L_2 \le \ldots \le L_{2d}$.

Conjecture. Fix $d \ge 3$. Each L_i is either $O(1)$ or $\Omega(N^d)$. More precisely, for $\omega_N \to \infty$ arbitrarily slowly,

$$P(\text{ some } L_i \in (\omega_N, N^d/\omega_N)) \to 0 \text{ as } N \to \infty.$$

The conjecture arises by analogy with simple RW on the torus, whose excursion lengths are either $O(1)$ or $\Omega(N^d)$. For $d = 2$ the analogy suggests that the short lengths in the random Eulerian circuit will be only $O(\log N)$ instead of $O(1)$.