INTRODUCTION TO SYMPLECTIC GEOMETRY AND TOPOLOGY
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Lecture 1

I. Introduction and Motivation

- Definitions and examples
- Motivating questions
- Remarkable results

II. Linear Symplectic Geometry

- Bilinear forms
- Subspaces and splittings
- The symplectic group

Lecture 2

IV. Complex Structures

- Compatible complex structures
- Symplectic and complex vector bundles

V. Examples of Symplectic Manifolds

- Surfaces
- Spheres only in dimension two
- Kähler manifolds
- Cotangent bundles

VI. Lack of Local Invariants (no proofs)

- Darboux's Theorem
- Neighborhood theorems
- Moser's method

Lecture 3

VII. Constructions of Symplectic Manifolds

- Hamiltonian circle actions and symplectic reduction
- Blowing up and down
- Symplectic cutting and symplectic sum
- Fibrations and Lefschetz fibrations

Lecture 4

VIII. Gompf's Theorem about Fundamental Groups

- Idea, smooth construction
- Comment on symplectic aspects

IX. Origins: Hamiltonian Mechanics

- Hamilton's equations
- Harmonic oscillator
- Integrable systems
- Toric manifolds

Lecture 5

X. Toric Geometry

- Moment maps and convexity
- Delzant's Theorem
- Reading moment polytopes

Primary reference:

Introduction to Symplectic Topology, 2nd ed., McDuff and Salamon. Oxford Science Publications, 1998.