# Portraying and remembering

# Irving Kaplansky
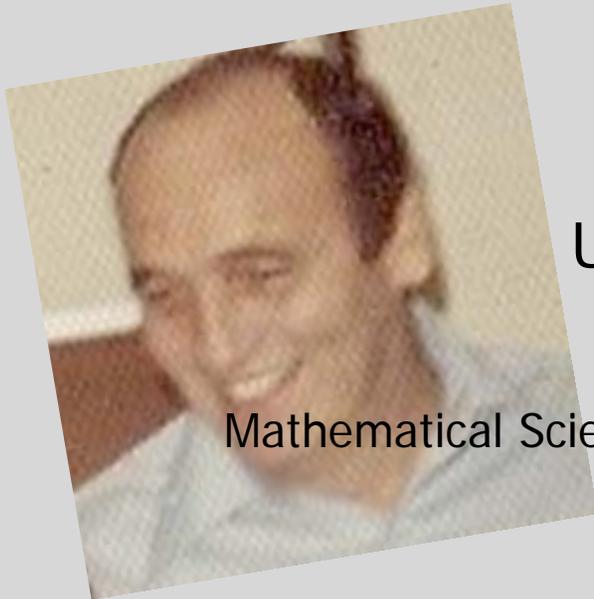
## Hyman Bass
### University of Michigan
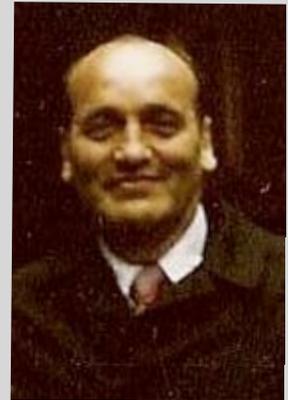
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# Irving ("Kap") Kaplansky

## "infinitely algebraic"

*"I liked the algebraic way of looking at things. I'm additionally fascinated when the algebraic method is applied to infinite objects."*

# 1917 - 2006

## A Gallery of Portraits

# Family portrait:  Kap as son

• Born 22 March, 1917 in Toronto, (youngest of 4 children) shortly after his parents emigrated to Canada from Poland.

• Father Samuel:  Studied to be a rabbi in Poland; worked as a tailor in Toronto.

• Mother Anna: Little schooling, but enterprising: "Health Bread Bakeries" supported (& employed) the whole family

Kap's father's grandfather



Kap's father's parents



Kap (age 4)  with family

# Family Portrait:  Kap as father

- 1951: Married Chellie Brenner, a grad student at Harvard

   Warm hearted, ebullient, outwardly emotional (unlike Kap)

- Three children:   Steven,   Alex,   Lucy

   *"He taught me and my brothers a lot, (including) what is really the most important lesson: to do the thing you love and not worry about making money."*

- Died 25 June, 2006, at Steven's home in Sherman Oaks, CA

   Eight months before his death he was still doing mathematics. Steven asked,

   - *"What are you working on, Dad?"*
   - *"It would take too long to explain."*

# Kap & Chellie marry 1951
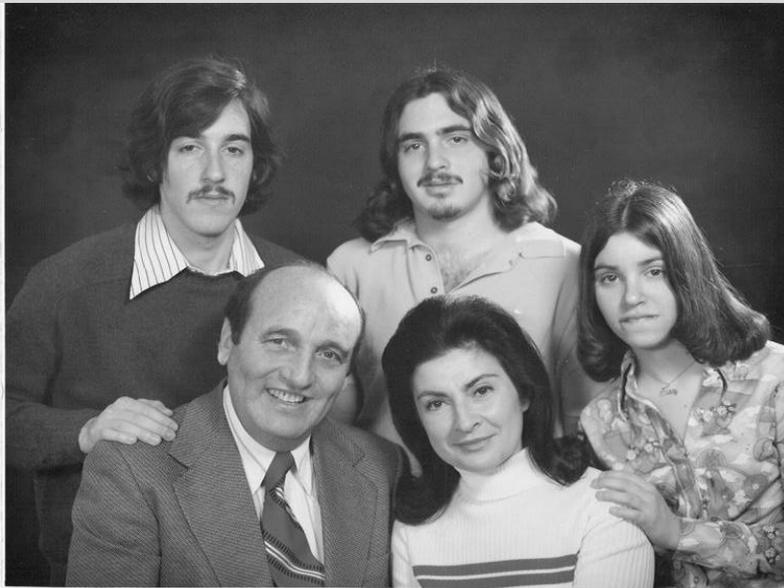


# Family portrait, 1972
Alex    Steven    Lucy
Kap    Chellie

# Kap – The perfect accompanist

"At age 4, I was taken to a Yiddish musical, *Die Goldene Kala*. It was a revelation to me that there could be this kind of entertainment with music. When I came home I sat down and played the show's hit song. So I was rushed off to piano lessons. After 11 years I realized there was no point in continuing; I was not going to be a pianist of any distinction."

"I enjoy playing piano to this day. ... God intended me to be the perfect accompanist – or better, the perfect rehearsal pianist. I play loud, I play in tune, but I don't play very well."

In HS: Dance bands. At Harvard: Small combo, Harvard jazz band. "Kaplansky Kapers" on Harvard radio station. ... Tom Lehrer was a student of mine, but I don't have his talents.

At U Chicago: Regular rehearsal pianist - Gilbert & Sullivan, caliope for football entertainment.

In Berkeley: Freight & Salvage Coffee House; once on "West Coast Live."

In later years, occasionally accompanied his daughter Lucy on tour.

# Kap at the keys
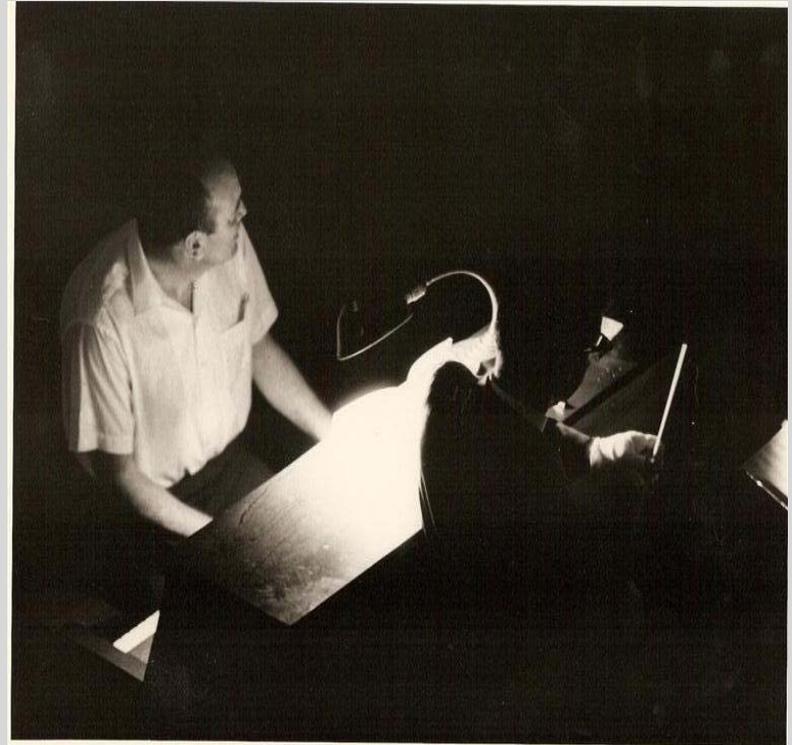




*From as early as I can remember I would sing while he played the piano. He taught me dozens of songs from the 1930's and 40's, as well as from Gilbert and Sullivan operettas. I still remember most of these songs.* (Lucy)

# Kap's  Song About π

Golden age of song, ~ 1920-1950 (pre-rock & roll, ...).

Most had the form AABA.  I noticed there was a second form ("Type 2") AA'BAA'BA".  A: 4 bar theme; A', A" : variants; B: contrasting 8 bar theme. (Though I assumed any jazz musician knew about this, nothing about it was found in the literature.) *Type 2 is really better for songs*. (In Woody Allen's *Radio Days*, the majority of the 20 songs are Type 2.)  As proof I tried to show that you could make a passable song out of such an unpromising source of thematic material as the first 14 digits of π.

Enid Rieser produced lyrics.  Lucy Kaplansky often performs this on her tours.

# SONG ABOUT π

In all the bygone ages,
Philosophers and sages
Have meditated on the circle's mysteries.
From Euclid to Pythagoras,
From Gauss to Anaxag'ras,
Their thoughts have filled the libr'ies bulging histories.
And yet there was elation
Throughout the whole Greek nation
When Archimedes made his mighty computation!
He said:

CHORUS

3 1 41 Oh (5) my (9), here's (2) a (6) song (5) to (3) sing (5) about (8,9) pi (7).
Not a sigma or mu but a well-known Greek letter too.
You can have your alphas and the great phi-betas, and omega for a friend,
But that's just what a circle doesn't have--a beginning or an end.
3 1 4 1 5 9 is a ratio we don't define;
Two pi times radii gives circumf'rence you can rely;
If you square the radius times the pi, you will get the circle's space.
Here's a song about pi, fit for a mathematician's embrace.

# Kap's career

| | | |
|---|---|---|
| 1938 | B. A. | U Toronto, |
| 1939 | M. A. | U Toronto |
| 1941 | Ph. D. | Harvard |
| 1941-44 | Benjamin Pierce Instructor, Harvard U | |
| 1944-45 | Applied Mathematics Group, Columbia U [*] | |
| 1945-84 | Mathematics Department, U Chicago | |
| 1962-67 | Department Chair | |
| 1984-92 | Director, MSRI | |
| 1985-86 | President, AMS | |

[*] Brought there by MacLane, for defense work: *"So that year was spent largely on ordinary differential equations. I had a taste of real life and found that mathematics could actually be used for something."*

# First years of the Putnam Competition

**Putnam fellows** included:

| | | **Team Winner** |
|---|---|---|
| 1938 | **Irving Kaplansky** [(*)] | |
| | George Mackey | 1938  Toronto |
| 1939 | Richard Feynmann | |
| 1940 | Andrew Gleason | 1939  Brooklyn College |
| 1941 | Andrew Gleason | 1940  Toronto |
| | Richard Arens | 1941  Brooklyn College |
| 1942 | Andrew Gleason | |
| | Harvey Cohn | 1942  Toronto |
| | WWII | |
| 1946 | Felix Browder | 1946  Toronto |
| | Eugenio Calabi | |
| | Maxwell Rosenlicht | |

----------------------------------------

[(*)] Senior, U. Toronto; First Putnam Fellow, at Harvard

# The "Stone Age" at Chicago

1945    **Kap** arrives

1946    **Marshall Stone** arrives to build Dept; four gigantic appointments:

**Saunders MacLane**          **Antoni Zygmund**

**André Weil**                      **Shiing-Shen Chern**

Plus waves of younger people

Influential younger colleagues:

Irving Segal            Paul Halmos            Ed Spanier

# Austere regularity, and swimming

- He scheduled classes & meetings <u>early!</u>

- Swimming: (Lake Michigan shore - several hours)
  A lifetime habit.

- Lunch on the fly.  Little social life before his marriage.

- "*Dad taught me to be organized in everything, reliable, and punctual. I think I'm the only musician I know who always shows up on time and actually does what I say I'm going to do.*"  (Lucy Kaplansky)

- Popular with grad students, always ready to talk math, but very focused, no 'small' talk.      *"Cut the crap. Let's talk mathematics."*

# Kap the administrator

1962-67:    **Chair of U Chicago Department of Mathematics**

1968-72:    Member AMS Board of Trustees,
  1971-72:  Chair

1969-71:    Vice President of AMS

1984-92:    **Director of MSRI**

1985-86:    **President of AMS**

1990-94:    Member of Council of American Academy of Arts & Sciences.

1st Putnam Fellow, Harvard, ~1940



Chair, U Chicago Math Dept, 1962-67



IRVING KAPLANSKY, PRESIDENT, 1985-1986

President AMS, 1985-86



Director MSRI, 1984-92

UNIVERSITY of TORONTO

Harvard University

THE UNIVERSITY OF CHICAGO

MSRI
Mathematical Sciences Research Institute

AMERICAN MATHEMATICAL SOCIETY

# Kap the research mathematician

## Broad Areas of Research

| | |
|---|---|
| **TA** | **Topological algebra, operator algebras, etc.** |
| **Q** | **Quadratic and other forms, both arithmetic and algebraic theory** |
| **C** | **Commutative and homological algebra** |
| **R** | Ring theory, including differential algebra |
| **Lie** | **Lie theory, including infinite dimensional** |
| **#** | Combinatorics, and some number theory |
| **M** | Module theory, including abelian groups |
| **L** | Linear algebra |
| **G** | General, including general algebra, group theory, game theory |
| **PS** | Probability and statistics |

# Kap's publication profile

| Putnam U Toronto '38 Harvard PhD '41 | WWII work '44-'45 @Columbia U | University of Chicago '45 – '84  Department Chair '62 - '67 | | | | | | Pres. AMS '85 – '86  Director MSRI '84 – '92 | | | Retired | |

**Legend:**

- **TA** — Topological algebra, operator algebras, etc.
- **Q** — Quadratic and other forms, both arithmetic and algebraic theory
- **C** — Commutative and homological algebra
- **R** — Ring theory, including differential algebra
- **Lie** — Lie theory, including infinite dimensional
- **#** — Combinatorics, and some number theory
- **M** — Module theory, including abelian groups
- **L** — Linear algebra
- **G** — General, including general algebra and group theory
- PS — Probability and statistics
- **B** = book

| # | < '45 | '45-'49 | '50-'54 | '55-'59 | '60-'64 | '65-'69 | '70-'74 | '75-'79 | '80-'84 | '85-'89 | '90-'94 | '95-'99 | '00-'06 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 30 | | PS | | | | | | | | | | | |
| 29 | | PS | | | | | | | | | | | |
| 28 | | PS | | | | | | | | | | | |
| 27 | | M | **B** (M) | | | | | | | | | | |
| 26 | | M | M | | | | | | | | | | |
| 25 | | R | M | | | | | | | | | | |
| 24 | | R | R | | | | | | | | | | |
| 23 | | R | R | | | | | | | | | | |
| 22 | | R | Q | | | | | | | | | | |
| 21 | | G | Q | | | | | | | | | | |
| 20 | | C | Q | | | | | | | | | | |
| 19 | | # | Q | | | | | | | | | | |
| 18 | | # | TA | | | | | | | | | | |
| 17 | | # | TA | | | | | | | | | | |
| 16 | | # | TA | | | | | | | | | | |
| 15 | | # | TA | | | | | | | | | | |
| 14 | | # | TA | | | | | | | | | | |
| 13 | | TA | TA | | | | | | | | | | |
| 12 | | TA | TA | | | | | | | | | | |
| 11 | | TA | TA | | | | **B** (GTop) | Lie | | | | | |
| 10 | | TA | TA | | | | P | Lie | | | | L | |
| 9 | | TA | TA | | | B (TA) | L | Lie | | | | Q | |
| 8 | | TA | TA | | | **B** (L) | Lie | Lie | | | G | Q | |
| 7 | | TA | TA | **B** (DAlg) | | **B** (M) | Lie | Lie | | G | G | Q | |
| 6 | | TA | TA | Lie | | M | Lie | R | G | TA | TA | Q | |
| 5 | PS | TA | TA | M | **B** (Lie) | **B** (R) | P (R) | R | Lie | Lie | Lie | Q | |
| 4 | PS | TA | TA | C | C | C | **B** (C) | R | Lie | Lie | Lie | Q | |
| 3 | # | TA | TA | R | C | Q | C | R | L | R | L | Q | G |
| 2 | R | TA | TA | TA | C | Q | C | **B** (C) | R | R | R | Q | Q |
| 1 | R | TA | TA | B (TA) | C | Q | Q | M | R | R | Q | Q | Q |

**GRAPH OF KAPLANSKY'S PUBLICATIONS**

# Comments on Kap's publication profile

- <u>WWII inventory</u>? Early papers in statistics, combinatorics, game theory
- <u>Topological algebra</u> 'explosion' (32 papers in 1948-52!). Leading edge of the field. Reviewers: Dieudonné, Godement, Dixmier (Bourbaki), ... This is the 'mountain' in Kap's publication profile.          (Dick Kadison will say more)
- Distill the algebraic essence, curfew on the analysis. Favorite paper: *"Any orthocomplemented complete modular lattice is a continuous geometry."*
- <u>Ring theory</u>: Most influential paper, "Rings with polynomial identity." Opened a whole new field.     Kurosh (ring analogue of Burnside) problem.
- <u>Lie theory</u>: Hilbert's Fifth Problem. Characteristic p, infinite dimensional structure theory, connections with physics.
- <u>Quadratic (& higher) forms</u>: Dear to Kap's heart, both abstract, and (in later years) concrete number theoretic.
- <u>Commutative and homological algebra</u>: The area Kap is most identified with in the eyes of many. Yet the publication profile shows that this is a relatively small part of his published oeuvre. How can that be??
- Kap's "student profile" furnishes an answer.

# Kap's student profile

## Kaplansky had 55 students and 617 descendants

| | < '45 | '45-'49 | '50-'54 | '55-'59 | '60-'64 | '65-'69 | '70-'74 | '75-'79 | '80-'84 | '85-'89 | '90-'94 | '95-'99 | '00-'04 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | | | | | | Richard Tarcy '69 | | | | | | | |
| 14 | | | | | | Robert Kibler '69 | | | | | | | |
| 13 | | | | | | Edward Evans, Jr. '69 | | | | | | | |
| 12 | | | | | | Richard Wagner '68 | | | | | | | |
| 11 | | | | Günther Lumer '59 | | Richard Shaker '68 | | | | | | | |
| 10 | | | | Hyman Bass '59 | | Howard Gorman '68 | | | | | | | |
| 9 | | | Jacob Feldman '54 | Joseph Rotman '59 | | Siusanna Epp '68 | | | | | | | |
| 8 | | | Fred Wright, Jr. '53 | Eben Matlis '58 | Jacob Towber '64 | Mahlon Michael Day '67 | Charles Hanna '74 | | | | | | |
| 7 | | | Oscar Litoff '53 | Donald Ornstein '57 | Ronald Hamelink '64 | Alphonse Buccino '67 | Daniel Anderson '74 | | | | | | |
| 6 | | | William Darsow '53 | Hazleton Mirkilmozrw '57 | Fred Richman '63 | Wolmer Vasconcelos '66 | Jacob Matijevic '73 | | | | | | |
| 5 | | | H. Arlen Brown '52 | Edeward Posner '57 | Kolumum Nagarajan '62 | Dean Heller '66 | Adrian Wadsworth '72 | | | | | | |
| 4 | | | Isidor Fleisher '52 | George Kolettis, Jr. '57 | Robert MacRae '61 | Hwa Tsang '65 | Peter Kohn '72 | | | | | | |
| 3 | | | Alex Rosenberg '51 | Sterling Berberian '55 | Edward Davis '61 | Gerson Levin '65 | Judith Sally '71 | Harry Hutchins '78 | | | | | |
| 2 | | | Flora Dinkines '51 | Harold Widom '55 | John Eagon '61 | Yung-yung Kuo '65 | Brice Prekowitz '71 | Michael Modica '75 | | | | | |
| 1 | | | Chester Feldman '50 | Malcolm Goldman '55 | Steven Chase '60 | Samuel Gedwiser '65 | Stephen McAdam '70 | Warren Nichols '75 | | | | | |

# What do we notice?

- Two measures of mathematical productivity: publications        students.

- The relative 'masses' of topological algebra and of commutative and homological algebra are reversed. And notice also the time shift.

- Kap was a pioneer and major developer of topological algebra.

- In commutative and homological algebra, he was a learner and apprentice teacher (of apprentices).

# Commutative & homological algebra, 50s-60s

**Background currents**

- <u>Homological algebra</u> -> category theory (Cartan, Eilenberg, MacLane, Grothendieck, ...)  First developed by algebraic topologists, not algebraists.
- Serre-Grothendieck refounding of <u>algebraic geometry</u>, with expanded foundations in <u>commutative algebra</u>

**Where Kap enters**

- New algebraic tool for ring theory:  <u>homological dimension</u>.  What is its algebraic significance?
- Breakthrough:  For a (commutative) noetherian local ring,

    finite global homological dimension   <=>   regular (non-singular)  (Auslander-Buchsbaum-Serre) and

    homological formulation of intersection multiplicities
- Work known only on the Cambridge (MA) - Paris axis.
- Kap offered courses on these developments, still in motion, and lifted a whole generation of young researchers (myself included) into this space
- Use of these ideas to prove unique factorization for regular local rings  (A-B).
- This played out for Kap over the next two decades, with students and books to show for it.

# Kap the Teacher & Mentor

- "*I like the challenge of organizing my thoughts and trying to present them in a clear and useful and interesting way. On the other hand, to see the faces light up, as they occasionally do, to even get them excited so that maybe they can do a little mathematical experimentation themselves – that's possible, on a limited scale, even in a calculus class.*"

- Advice to students: "*Look at the first case, the easiest case that you don't understand completely. Do examples, a million examples, "well chosen" examples, or "lucky" ones. If the problem is worthwhile, give it a good try – months, maybe years if necessary. Aim for the less obvious, things that others have not likely proved already.*"

- And: "*Spend some time every day learning something new that is disjoint from the problem on which you are currently working (remember that the disjointness may be temporary). And read the masters.*"

- "*When a great mathematician has mastered a subject to his satisfaction and is presenting it, that mastery comes through unmistakably, so you have an excellent chance of understanding quickly the main ideas.*" [He cites Weil, Serre, Milnor, Atiyah.]

- "*. . . the thing that bedevils the mathematical profession – the difficulty we have in telling the world outside mathematics what it is that mathematicians do. And for shame, for shame, right within mathematics itself, we don't tell each other properly.*"

# As seen by others

- *He was not only a fantastic mathematician but a marvelous lecturer, and he had a remarkable talent for getting the best out of students.*      – **Dick Swan**

- *Every course, indeed, every lecture, was a delight. Courses were very well-organized, as was each lecture. Results were put in perspective, their applications and importance made explicit. Humor and droll asides were frequent. Technical details were usually prepared in advance as lemmas so as not to cloud the main ideas in a proof. Hypotheses were stated clearly, with examples showing why they were necessary. The exposition was so smooth and exciting, I usually left the classroom feeling that I really understood everything. To deal with such arrogance, Kap always assigned challenging problems, which made us feel a bit more humble, but which also added to our understanding. He was a wonderful teacher, both in the short term and for the rest of my mathematical career. His taste was impeccable, his enthusiasm was contagious, and he was the model of the mathematician I would have been happy to be.*      – **Joe Rotman**

- *I did know about the work of Emmy Noether and it may have influenced my choice of area, algebra, although I think the teaching of Irving Kaplansky was what really inspired me*      – **Vera Pless**

- *I was interested in this, and having reached what Irving Kaplansky calls the age of ossification when the only way to learn something new is to teach it, I gave a graduate course on this work.*      – **Edward Nelson**

# Kap's mathematical taste & style

- Kap was a problem solver of great virtuosity.  He sought problems, and theorems of great pedigree, and probed them deeply.

- His main focus was on proofs (pathways), more than on theorems (destinations).  He sought geodesics, and the most economic (high mileage) means to get there.

- Proof analysis led to double edged kinds of generalization/axiomatization:

  - A given proof yields more than claimed. The given hypotheses deliver more than the stated theorem promises.
  - The hypotheses can be weakened.  We can get the same results more cheaply.

- The strength of this disposition was perhaps sometimes over zealous, pushing toward "premature maturation" of the mathematics.

- But it was a very powerful mode of instruction, yielding deep conceptual command of the territory covered.

# Re-Kap

A man of many admirable qualities –
disciplined, focused, dedicated, creative, nurturing

precocious student

talented and expressive musician

loving and nurturing husband and father

creative and prolific research mathematician

inspiring teacher and mentor of a generation of researchers

leader of institutions and of the professional community

*A LIFE TO BE CELEBRATED*