

## Cohomological approaches to rational points

MSRI workshop 27-31 March 2006

MONDAY

9:00-9:30 Welcome (David Eisenbud and Staff)

9:30-10:30

Minhyong Kim

University of Arizona

Title : Non-abelian cohomology varieties and Diophantine geometry

Abstract : We will discuss how non-abelian cohomology varieties associated to unipotent fundamental groups can be used to study rational points on hyperbolic curves.

10:30-11:00

Tea break

11:00-12:00

Gaël Rémond

Institut Fourier, Grenoble

Title : On a conjecture of Pink

Abstract : Let  $A$  be a semi-abelian variety,  $X$  a subvariety of  $A$  and  $S$  the intersection of  $X$  with the union of all algebraic subgroups of  $A$  of codimension greater than the dimension of  $X$ . One conjectures that if  $X$  is not contained in any proper algebraic subgroup of  $A$  then the set  $S$  is not Zariski dense in  $X$ . I discuss several results in the direction of this statement, in particular when  $S$  is replaced by its subset of points of sufficiently large height.

Lunch break

2:15-3:15 Shou-Wu Zhang

Columbia University

Title : Distributions in arithmetic dynamics

Abstract : In this talk I will introduce arithmetic dynamics and describe some questions about the distributions of rational points, preperiodic points, and small points with respect to various topologies: dynamic topology, Zariski topology, adelic topology, and distributional topology.

3:15-3:45 Tea break

3:45-4:45 Carlo Gasbarri

Università Tor Vergata, Roma

Title : Dyson's Theorem for the product of two curves

Abstract : We shall speak on an analogue of Dyson's theorem for the product of two curves. This gives a self-contained, new proof of Siegel's theorem on integral points on hyperbolic curves. This proof does not rely upon Roth's theorem, the Mordell-Weil theorem or linear forms in logarithms. In the final part of the talk effectiveness will be discussed.

TUESDAY

9:30-10:30

Hélène Esnault

Universität Duisburg-Essen

Title : Rational points over finite fields and cohomological methods

Abstract : We will give a survey of the various cohomological methods we recently developed in order to find congruences modulo  $q$ -powers for the number of rational points of algebraic varieties defined over  $F_q$ .

10:30-11:00

Tea break

11:00-12:00

János Kollár

Princeton University

Title: A conjecture of Ax and degenerations of Fano varieties

Abstract : James Ax conjectured that every pseudo algebraically closed field is  $C_1$ . We prove this conjecture in characteristic 0 by relating it to degenerations of Fano varieties.

Lunch break

2:15-3:15

Jason Starr

Massachusetts Institute of Technology

Title: Rational 1-connectedness and rational points

Abstract: Every separably rationally connected variety over the function field of a curve has a rational point. Conjecturally, under some additional hypotheses, a rationally 1-connected variety over the function field of a char 0 surface has a rational point. I will discuss this conjecture, a theorem, and a corollary: a new proof of de Jong's period-index theorem unconditional on the characteristic. This is joint work with de Jong. Time permitting, I will also discuss connections to weak approximation over function fields of curves.

3:15-3:45

Tea break

3:45-4:45

Max Lieblich

Princeton University

Title : Period and index in the Brauer group of a relative curve

Abstract : I will present an approach to studying period-index problems for Brauer groups of function fields which relates the properties of Brauer classes to the existence and degrees of points on certain rational varieties. Specific applications include a new proof of de Jong's period-index theorem for function fields of algebraic surfaces and a conditional proof of an analogous result for function fields of arithmetic surfaces, assuming certain instances of the standard local-to-global conjectures for smooth rational projective varieties over global fields.

WEDNESDAY

9:30-10:30

Shuji Saito

Nagoya University, Chikusa-ku

Title : Weak Bloch-Beilinson conjecture for zero cycles over  $p$ -adic fields. (joint work with Kanetomo SATO)

Abstract: Let  $V$  be a smooth projective variety over a  $p$ -adic field  $k$ . Let  $CH_0(V)$  be the Chow group of the zero-cycles on  $V$  modulo rational equivalence with  $A_0(V) \subset CH_0(V)$ , the subgroup of cycle classes of degree 0. The main theorem of the talk is the following:

Theorem 1: Assume that  $V$  has a regular projective flat model  $X$  over the ring of integers in  $k$  such that the reduced part of its special fiber is a simple normal crossing divisor on  $X$ . Then  $A_0(V)$  is the direct sum of a finite group and the maximal  $p'$ -divisible subgroup of  $A_0(V)$ . Here an abelian group is  $p'$ -divisible if it is divisible by any integer prime to  $p$ .

The above theorem is deduced from the following theorem:

Theorem 2: Let  $X$  be as above with  $d + 1 = \dim(X)$ . For an integer  $n > 0$  prime to  $p$ , the étale cycle map for 1-cycles on the model  $X$

$$\rho_X : CH_1(X)/n \rightarrow H_{\text{ét}}^{2d}(X, \mu_n^{\otimes d})$$

is an isomorphism.

10:30-11:00

Tea break

11:00-12:00

Alexander Merkurjev

University of California at Los Angeles

Title : Zero-cycles on homogeneous varieties

Abstract : Let  $X$  be a projective homogeneous variety over a field  $F$ . We study the subgroup  $C(X) \subset CH_0(X)$  of degree zero 0-dimensional cycles on  $X$ . We introduce an equivalence relation on the class of all finite field extensions  $L/F$  with  $X(L) \neq \emptyset$  and use this relation to show that  $C(X) = 0$  for certain classes of  $X$ . We also give examples of  $C(X) \neq 0$ . (This is joint work with V.Chernousov.)

Lunch break

2:15-3:15

Emmanuel Peyre

Université Joseph Fourier (Grenoble) and MSRI

Title : Height zeta functions : an arithmetic-motivic dictionary

Abstract : The analogy between number fields and function fields of curves is a truism of arithmetic geometry. It is therefore natural to look for links between the asymptotic behaviour of points of bounded height on a variety and the geometry of the moduli space of morphisms of high degree from a curve to this variety. Height zeta functions provide a working dictionary between the arithmetic and motivic settings. This opens a trail toward a better understanding of the geometric obstructions for the Batyrev-Manin's principle.

3:15-3:45

Tea break

3:45-4:45

David Bourqui

IRMAR, Université Rennes I

Title : Height zeta functions : the case of toric varieties

Abstract : We will sketch the computation of the height zeta function of toric varieties defined over a global field of arbitrary characteristic. The talk will focus on the main term of the height zeta function, whose value is somewhat unexpected in positive characteristic. If time allows, we will also say a few words about the motivic case.

Wednesday, 5 to 6 : reception

THURSDAY 9:30-10:30

Michael Stoll

International University Bremen

Title : Finite descent obstructions and rational points

Abstract : We will look at the information on the location of the rational points inside the adelic points on a smooth projective variety  $X$  over a number field that can be obtained via  $X$ -torsors under finite  $k$ -group schemes. We will compare this to the Brauer-Manin obstruction. If  $X$  is a curve, the results are particularly nice, leading to the conjecture that on a curve of higher genus, the information coming from finite abelian torsors cuts out precisely the rational points. Some evidence for this conjecture will be presented.

10:30-11:00 Tea break

11:00-12:00

Olivier Wittenberg

Université Paris-Sud and MSRI

Title : Rational points on pencils of curves of genus 1

Abstract : In 1993 Swinnerton-Dyer obtained the first non-trivial results on the existence of rational points for surfaces endowed with a pencil of curves of genus 1 (over a number field). Since then, the methods used in his proof have been extended and refined by several authors, thereby leading to significant progress towards several classical arithmetic conjectures. In this talk I will attempt to describe the ideas that constitute the core of the technique.

Lunch break

2:15-3:15

Philippe Gille

CNRS, Université Paris-Sud

Title : Rational points on homogeneous spaces : a survey

Abstract : After an introduction to this classical topic, we shall discuss Hasse principle and weak approximation properties for homogeneous spaces defined over geometric fields of dimension 1 and 2. (Joint works with J.-L. Colliot-Thélène and R. Parimala.)

3:15-3:45 Tea break

3:45-4:45

David Harari

Université Paris-Sud

Title : 1-motives and principal homogeneous spaces of algebraic groups

Abstract : Let  $G$  be a (not necessarily linear) connected algebraic group. We show that the Brauer-Manin obstruction to the Hasse principle and to weak approximation is the only one for principal homogeneous spaces of  $G$ . When  $G$  is a semi-abelian variety, we prove (joint work with T. Szamuely) a more precise result thanks to arithmetic duality theorems for 1-motives

FRIDAY

9:30-10:30

Alexei Skorobogatov

Imperial College, London and MSRI

Title : The elementary obstruction for homogeneous spaces

Abstract : If  $X$  is a smooth, geometrically integral variety over a field  $k$  with algebraic closure  $\bar{k}$  such that the natural inclusion of Galois modules  $\bar{k}^* \rightarrow \bar{k}(X)^*$  has no section, then  $X$  has no  $k$ -points. We prove that the converse is also true if  $k$  is a finite extension of the field of  $p$ -adic numbers, and  $X$  is a homogeneous space of a (not necessarily affine) connected algebraic group with connected stabilizers. We construct an example which shows that the same statement over the field of rational numbers does not hold in general. (Joint work with Borovoi and Colliot-Thélène.)

10:30-11:00

Tea break

11:00-12:00

Karl Rubin

University of California at Irvine and MSRI

Title: Growth of ranks of abelian varieties in dihedral extensions

Abstract: In joint work with Barry Mazur, we obtain lower bounds for Selmer ranks of abelian varieties over dihedral extensions of number fields. Except in very special cases where one has a large family of Heegner points, the source of these Selmer classes is very mysterious.

End of the workshop