Superbosonization of invariant matrix ensembles

Abstract:

Superbosonization is a new variant of the method of commuting and anticommuting variables. The object of departure of the supersymmetry method is the Fourier transform of the probability measure of the given ensemble of disordered Hamiltonians. This Fourier transform is evaluated on a supermatrix built from commuting and anti-commuting variables and thus becomes a superfunction; more precisely a function f, which is defined on a complex vector space V_0 and takes values in the exterior algebra ΛV_1^* of another complex vector space V_1. If the probability measure is invariant under a group K, so that the function f is equivariant with respect to K acting on V_0 and V_1, then a standard result from invariant theory tells us that f can be viewed as the pullback of a superfunction F defined on the quotient of $V = V_0 \oplus V_1$ by the group K. The heart of the superbosonization method is a formula which reduces the integral of f to an integral of the lifted function F .

In the talk will be concerned with the algebra $\mathcal{A}^G(V)$ of G-equivariant holomorphic functions

$$f : V_0 \to \Lambda(V_1^*), v \mapsto f(v) = g.f(g^{-1}v) \quad (g \in G),$$

for the classical Lie groups $G = GL_n(\mathbb{C}), O_n(\mathbb{C})$, and $Sp_n(\mathbb{C})$, and the vector spaces

$$V_0 = \mathrm{Hom}(\mathbb{C}^n, \mathbb{C}^p) \oplus \mathrm{Hom}(\mathbb{C}^p, \mathbb{C}^n), V_1 = \mathrm{Hom}(\mathbb{C}^n, \mathbb{C}^q) \oplus \mathrm{Hom}(\mathbb{C}^q, \mathbb{C}^n).$$

Our strategy is to lift $f \in \mathcal{A}^G(V)$ to another algebra $\mathcal{A}(W)$ of holomorphic functions $F : W_0 \to \Lambda(W_1^*)$, using a surjective homomorphism $\mathcal{A}(W) \to \mathcal{A}^G(V)$. The aim is then to show a statement of reduction – the superbosonization formula – transferring the Berezin superintegral of $f \in \mathcal{A}^G(V)$ to such an integral of $F \in \mathcal{A}(W)$.

References

[1] P. Littelmann, H.-J. Sommers, M.R. Zirnbauer, Superbosonization of invariant matrix ensembles, preprint arXiv:0707.2929v1 [math-ph] (2007).