Critical Issues in Education: Teaching and Learning Algebra

Mathematical Sciences Research Institute, Berkeley, California
May 14 – May 16, 2008

Wednesday, May 14, 2008

Charter Bus Scheduled Departures	1:45 PM Depart Double Tree Hotel 2:00 PM Depart Hearst Mining Circle	
	UC Berkeley "Hill" shuttle departs UC Berkeley's Hearst Mining Circle every half hour beginning at 7:40AM	
2:00 – 2:30PM		*Coffee, tea in the* ***Atrium***
	Session 1.1	*Plenary Session in the* ***Simons Auditorium***
2:30– 3:00PM	Robert Bryant, MSRI Director Deborah Ball, University of Michigan	Welcome, Overview, and Purpose of Workshop/Framing Questions
Question 1	**What are some organizing principles around which one can create a coherent pre-college algebra program?**	
3:00-5:30PM	Al Cuoco, Center for Mathematics Education Diane Resek, San Francisco State University Tom Sallee, University of California, Davis Zalman Usiskin, University of Chicago James Fey, University of Maryland	Panel discussion (See Abstracts)
5:30-6:30PM		*Reception and light buffet dinner in the* ***Atrium***
	Session 1.2	*Plenary Session in the* ***Simons Auditorium***
6:45-7:15PM	Deborah Ball, University of Michigan	The National Mathematics Advisory Panel report: Summing Up and Taking Stock (See Abstract)
7:15-7:45PM	William McCallum, University of Arizona	Report on the NCTM *Lenses on High School Mathematics* report (See Abstract)
7:45-8:45PM	Hyman Bass, University of Michigan Roger Howe, Yale University	Discussants on the presentation
Charter Bus Scheduled Departures	9:00 PM Hearst Mining Circle, Double Tree Hotel	

Thursday, May 15, 2008

Charter Bus Scheduled Departures	7:00AM Depart Double Tree Hotel 7:15AM Depart Hearst Mining Circle
	UC Berkeley "Hill" shuttle departs UC Berkeley's Hearst Mining Circle every half hour beginning at 7:40 AM

7:30-8:00AM		*Coffee, tea in the **Atrium***
	Session 1.3	*Plenary Session in the **Atrium***
8:00-8:15AM	Al Cuoco, Center for Mathematics Education	Overview of the day.
8:15-9:15AM	*Parallel Sessions: Question 1*	
1.3a	Stephanie Ragucci, Core Plus/Andover High School Annette Roskam, CME/Rice Lake High School	***Simons Auditorium*** (See Abstract)
1.3b	Carol Cho, CPM/Alhambra High School Sybilla Beckmann, University of Georgia	***Baker Board Room*** (See Abstract)
1.3c	Matt Bremer, IMP/Berkeley High School Pat Thompson, Arizona State University	***Commons Room*** (See Abstract)
9:15-9:45AM		*Coffee, Tea, Danish, etc. in the **Atrium***
	Session 1.4	*Plenary Session in the **Simons Auditorium***
9:45-11:15AM	Roger Howe, Yale University William McCallum, University of Arizona Betty Phillips, Michigan State University	Discussants on Question 1.
11:15-1:00PM		*Lunch in the **Atrium***

Question 2 **What is known about effective ways for students to make the transition from arithmetic to algebra?**

	Session 2.1	*Plenary Session in the **Simons Auditorium***
1:00-3:00PM	David Carraher, TERC Jo Ann Lobato, San Diego State University Alan Schoenfeld, University of California, Berkeley Uri Treisman, University of Texas	Panel discussion (See Abstract)
3:00-3:30PM		*Coffee, Tea, Danish, etc. in the **Atrium***
3:30-4:30PM	*Parallel Sessions: Question 2*	

2.2a	Ted Courant, Bentley School Paul Goldenberg, CME	***Simons Auditorium*** (See Abstract)
2.2b	Virginia Bastable, Mount Holyoke College Susan Jo Russell, TERC Deborah Schifter, Education Development Center	***Baker Board Room*** (See Abstract)
2.2c	Betty Phillips, Michigan State University Mark Saul, Bronxville Schools (Ret.)	***Commons Room***
	Session 2.3	*Plenary Session in the* ***Simons Auditorium***
4:30-6:30PM	Hung-Hsi Wu, University of California, Berkeley Herb Clemens, chair, Ohio State University Robert Moses, The Algebra Project Paul Sally, University of Chicago	Question 2: The transition from arithmetic to algebra: further perspectives (See Abstracts)
Charter Bus Scheduled Departures	6:45PM Depart for Hearst Mining Circle and Double Tree Hotel.	

Friday, May 16, 2008

Charter Bus Scheduled Departure	7:00AM Depart Double Tree Hotel 7:15AM Depart Hearst Mining Circle UC Berkeley "Hill" shuttle departs UC Berkeley's Hearst Mining Circle every half hour beginning at 7:40 AM	
7:30-8:00AM		*Coffee, tea in the* ***Atrium***
	Session 2.4	*Plenary Session in the* ***Simons Auditorium***
8:00-8:15AM	James Fey, University of Maryland	Overview of the Day.
8:15-9:45AM	Hyman Bass, University of Michigan Megan Franke, University of California, Los Angeles Ed Silver, University of Michigan	Discussants on Question 2.
9:45-10:15AM		*Coffee, tea in the* ***Atrium***
Question 3	**What Algebraic understandings are essential for success in beginning collegiate mathematics?**	
	Session 3.1	*Plenary Session in the* ***Simons Auditorium***
10:15-11:45AM	William McCallum, IME/University of Arizona Tom Roby, University of Connecticut Deborah Hughs-Hallett, IME/University of Arizona	Panel discussion. See Abstracts.

11:45-12:45PM		*Lunch in the **Atrium***
12:45-1:45PM	*Parallel Sessions: Question 3*	
3.1a	William McCallum, University of Arizona Glenn Stevens, Boston University	***Simons Auditorium*** (See Abstract)
3.1b	Dan Chazan, University of Maryland James Fey, University of Maryland	***Baker Board Room*** (See Abstract)
	Session 3.2	*Plenary Session in the **Simons Auditorium***
1:45-3:15PM	Herb Clemens, Ohio State University Mark Saul, Bronxville Schools, ret. Ed Silver, University of Michigan	Discussants on Question 3.
3:15-3:45PM		*Coffee, tea in the **Atrium***
	Session 3.3	*Plenary Session in the **Simons Auditorium***
3:45-5:15PM	Dan Chazan, University of Maryland Al Cuoco, Center for Mathematics Education Hung-Hsi Wu, University of California, Berkeley	Panel: Preparing teachers to teach algebra (See Abstract)
5:15-5:45	Deborah Ball, University of Michigan	Closing Session: Connections among the questions
5:45-6:45PM		*Reception in the **Atrium***
Charter Bus Scheduled Departures	7:00PM Depart for Hearst Mining Circle and Double Tree Hotel.	

Abstracts

Question 1: What are some organizing principles around which one can create a coherent pre-college algebra program?

1. 1

Panel Abstract: There are several curricular approaches to developing coherence in high school algebra, each based on a framework about the nature of algebra, the ways in which students will use algebra in their post-secondary work, and how students learn. The presentations by this panel will describe several different ways of thinking about this question and representing those ideas in high school curriculum materials.

AL CUOCO
ALGEBRA IN THE CME PROJECT

The actual development of the CME Project began in 1992 with a grant from NSF to develop a geometry course, but the basic principles on which the program rests have evolved over close to four decades. Essentially, we've come to the position that the real utility of mathematics—both for students who specialize in mathematically related fields and for those who take other directions—lies in a style of work, a collection of mathematical habits of mind, that mathematicians use to make sense of the world. Such mathematical habits include visualization, performing thought experiments, reasoning by continuity or linearity, and mixing deduction with experiment. The basic results and methods of high school mathematics—the Pythagorean theorem, the techniques for solving equations or graphing lines or analyzing data—are the products of mathematics. The actual mathematics lies in the thinking that is used to create and develop these results. It is essential to develop both the results and the thinking. Our entire program, from our uses of technology to the design of the problem sets in each lesson, is devoted to helping students become mathematical thinkers as they develop the content knowledge to apply that mathematical thinking competently.

My presentation will give some examples of how this philosophy plays out in the context of the program's development of algebra in high school. There are several mathematical habits that are dominant in algebra: reasoning about and picturing calculations and operations, seeking regularity in repeated calculations, purposeful transformation and interpretation of algebraic expressions, "chunking" (changing variables in order to hide complexity), and seeking and modeling structural similarities in algebraic systems that include the number systems of arithmetic but also include other algebraic systems. I'll show how these habits are introduced and strengthened across algebra strand of the program, providing students with general-purpose tools that allow them to solve problems, make and use abstractions, and develop mathematical theories.

Diane Resek
Conceptualizing the Algebra Curriculum in IMP

There were three overriding goals that determined the organization scheme for algebra in the Interactive Mathematics Program curriculum. The first was to motivate students to engage with the material, especially students from underrepresented populations. To this end we wanted to interest students and to make the material accessible. A second goal was help them become powerful problem solvers, particularly when they would encounter unanticipated situations. Finally, we wanted to prepare them to become future learners of algebra and related fields both as university students and as participants in the global economy.

Tom Sallee
CPM (College Preparatory Mathematics Program)

The overriding goal of CPM is imparting long-term learning of the important mathematics in a way that students can use this knowledge in future courses and in life. To this end, we focus on: (1) teaching the most important topics thoroughly; (2) involving students in learning new mathematics in a way that connects this new knowledge with old; (3) teaching new ideas in a variety of real-world contexts whenever possible; and (4) reinforcing and extending knowledge by problems spaced over weeks or months.

Zalman Usiskin

Over the last four decades, the speaker has had the opportunity to create school algebra courses from the perspective of geometry and group theory. from the perspective of modeling, statistics, and applications, from the customary field postulates and theorems, and most recently from a perspective trying to take computer algebra systems into account. As a whole, these perspectives can change significantly the way one views the traditional subject matter of algebra, enriching the subject with theoretical connections, representations, and practical uses resulting in approaches quite different from that taken in the recent National Mathematics Advisory Panel report. (Zalman Usiskin)

James Fey
Developing High School Algebra Through Functions and Applications

In several experimental curriculum projects we have explored ways of developing key algebraic understandings and skills by starting with a focus on functions as models of relationships between variables in problem situations. I will present our rationale for this approach to high school algebra and some classroom experiences with it.

1.2

The National Mathematics Advisory Panel Report: Summing Up and Taking Stock

Deborah Ball

This presentation will offer a brief overview of the Panel's charge and process, a summary of the results, and comments appraising the outcomes.

Report on the NCTM Lenses on High School Mathematics report

William McCallum

In 2006 the NCTM released "Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics", a document intended to provide guidance for textbook authors and writers of state standards on areas of emphasis in the elementary and middle school grades. It is now working on a followup document for grades 9-12. I will discuss some of the challenges in writing this document and give a preliminary glimpse of some of its recommendations.

1.3a

Stephanie Ragucci and Annette Roskam

In this session, we will explore linear models and quadratic models using two different NSF-funded secondary mathematics curricula. First, we will experience how students learn to write equations for word problems in Algebra 1 using CME Project's Guess-Check-Generalize strategy. This strategy is a general-purpose tool that can be used throughout the entire program - to find equations for lines and curves, to find functions that agree with tables of data, and to express generality in repeated calculations. Then, we will move on to how Core-Plus Mathematics uses functions to help students create their own understanding of basic quadratic models. In this curriculum, students build a general quadratic function model by investigating the impact of the constant term, the linear term, and gravity. Both of these curricula offer a student-centered approach to learning, which we will examine using classroom examples.

1.3b

Carol Cho
This talk will exemplify the way that the Algebra Connections course of CPM continually requires students to connect a pattern or situation with numeric data (table), with the abstract (equation), and with the associated graph. The students are actively involved in their own original learning of new concepts as they constantly explain how the math is represented in each of the four modalities through real world problems or tile problems which they create. Spaced practice maintains and deepens the ideas with the difficulty level increasing as the book progresses.

Sybilla Beckman
Solving algebra story problems with simple "strip diagrams," solving them with algebra, and connecting the two approaches

Many nice algebra story problems can be solved by reasoning about very simple "strip diagrams" that show relationships among quantities. By drawing and reasoning about these diagrams, fourth, fifth, and sixth grade students can use arithmetic to solve algebra story problems without explicitly using algebra. This gives these students the opportunity to work on multi-step problems and to engage in analysis and thinking that is deeper than that required by straightforward arithmetic problems. Often, the arithmetic steps that these young students perform to solve the problems are essentially the same steps an older student would use in solving a linear equation (or a system of linear equations) to solve the problem. Connecting the strip diagram method to standard algebraic methods may be one way to help students make the transition from arithmetic to algebra.

1.3c

Session Overview

This session will introduce two complementary approaches to the design of instruction and curricula that support middle school and high school students' development of a mathematics that is useful in organizing their thinking about complex and sophisticated situations and ideas. "Algebra", then, is a natural and powerful aid to ways of thinking. It arises out of meanings that students make.

Matt Bremer

This session will give an example of how algebra develops in the Interactive Mathematics Program (IMP). In IMP activities are organized around large, complex problems. The details of one of those problem based units will be described and a video will show students at work on various aspects of the problem.

Pat Thompson

Quantitative reasoning, meaning the conceptualization of situations as constituted by things that have measures, can be developed in early grades and thereby be leveraged systematically in the design of algebra instruction and curricula. My presentation will focus on how two major algebraic activities grow naturally from quantitative reasoning: modeling quantitative relationships and reasoning about representational equivalence.

Question 2: What is known about effective ways for students to make the transition from arithmetic to algebra?

2.1

David Carraher
Mining the early mathematics curriculum

Many difficulties and much anguish in learning algebra in middle school may be an unintended consequence of how arithmetic (and, more generally early mathematics) has been taught. Various groups of mathematics educators are presently engaged in identifying overlooked opportunities for bringing out the algebraic character of arithmetic. I present several examples of such activities for grades 3-5 and show how they foreshadow and pave the way for concepts and skills that will take many years to master. I invite those present to reflect upon tensions between the concepts thought to be inherent to the activities and the evolving conceptualizations of students. We will also discuss the tension between modeling mathematical structure and modeling extra-mathematical phenomena

Jo Ann Lobato
*The Role of *Noticing* in the Transition to Algebra*

Understanding what allows children to successfully make the transition from arithmetic to algebra includes an investigation of how students come to notice particular mathematical properties when many sources of information compete for one's attention. What one notices in arithmetic contexts often forms the basis of what is generalized and symbolized as algebraic expressions. For the past three years, I have been examining how middle school students from different classes come to see situations involving quadratic and linear functions in distinctly different ways. Analyses of the classroom data demonstrate relationships between these different reasoning patterns and the nature of attention-focusing in each classroom. By building upon Chuck Goodwin s work on professional vision, I explore how what individuals notice in classrooms is socially organized and has ramifications for the nature of their algebraic reasoning.

Alan Schoenfeld
Why are Word Problems so Darned Hard?

In one sense, learning to solve word problems is like learning to ride a bicycle. After you've been doing it for a while it seems so easy that you wonder why anyone – including yourself – ever had trouble with it. But beginners do. For the past year I've been working in partnership with some middle school teachers, looking at their students' attempts to solve some early algebra word problems. From the student's perspective, the task is very complicated. There are linguistic issues, especially for second language learners; there are questions of what merits attention and what doesn't; there are questions of finding the right mathematical framing and representations for working through a problem, and also of getting clues from the mathematical organization of the problem itself. I'll unpack some of these complexities, which present great challenges to teachers, in my presentation.

Uri Treisman
Supporting the Transition to Algebra I: Emerging practices in large urban districts

As part of the work of the Urban Mathematics Leadership Network-a coalition of large urban districts focused on strengthening mathematics teaching and learning, the Dana Center has been systematically studying innovations in supporting students' transition to Algebra I. The resulting "practices worthy of attention" address such diverse issues as effective effort and the incorporation of advances in motivation and attribution theory to shaping students commitment to academic achievement, the development of academic language proficiency among English language learners, and supporting special education students in high school algebra classes. I'll provide glimpses into these innovations and briefly discuss strategies for bringing their use to scale.

2.2a
Paul Goldenberg
How the ideas and language of algebra K-5 set the stage for algebra 8-12

Some algebraic ideas precede arithmetic ones. Children's phenomenal language-learning capacity can build algebraic language just as well as any other that is "spoken" naturally in the child's environment, and we can tap that extraordinary as we teach children mathematics. These notions, and others, are part of our perspective on children's development of algebraic ideas and language, from Kindergarten sorting activities, through elementary school, culminating in knowledge that more than prepares them for success in formal algebra.

Ted Courant
Does 8th grade algebra prepare students for Geometry and high school mathematics?

2.2b
Deborah Schifter, Susan Jo Russell, and Virginia Bastable
Strengthening K-5 Arithmetic/Preparing for Algebra

About 15 years ago, a great deal of publicity was given to studies that indicated that secondary algebra courses were used to filter students, and that many students were excluded. Among the responses was to look at the elementary grades and consider how to better prepare students for algebra. Over the last decade and a half, there have been several teams studying what algebra in the elementary grades can be. Within this realm, the efforts of the presenters of this session have concentrated on two broad areas: 1) articulating, representing, and justifying generalizations about number and operations, and 2) investigating contexts of covariation with an emphasis on linear relationships. In this talk, we will present cases to illustrate what this work looks like in different classrooms at different grades, discuss how the content supports students' learning of core concepts at each grade, and demonstrate its importance in laying the conceptual foundation for the transition to algebra in middle school. In addition, we will discuss the importance of teacher learning.

2.2c
Betty Phillips
Transitioning from Arithmetic to Algebra

There have been many symposiums and proceedings over the past 15 years around the school algebra. Also, during this time several elementary, middle, and high school mathematics curricula have been developed that reflect different organizing themes for algebra. In this presentation I will share examples from a middle school curriculum that illustrates how situations from number, geometry, and data can provide a foundation for transitioning into an algebra curriculum that is organized around equivalence and patterns of change.

Mark Saul

Students making the transition from arithmetic to algebra are taking a large and important step in their mathematical development. This talk will examine this transition from arithmetic to algebra in the context of a trajectory of learning mathematics, from the early years through high school. Examples drawn from classroom practice will be used to examine the process of transition, from both a mathematical and a pedagogical standpoint.

2.3

Overall View
Herb Clemens, Ohio State University

This panel is charged with responding to, discussing and synthesizing the workshop presentations concerning the question: What is known about effective ways for students to make the transition from arithmetic to algebra?

The two hour session will be structured as follows: Each of three panelists will lead the session during 30 minutes. Each of the three 30-minute sections will be structured as follows: The panelist will be given ten minutes to comment on and synthesize what he gleaned from prior sessions on this topic at the workshop, followed by a 5-minute Q&A (in which the other panelists and audience are invited to participate), followed by ten minutes in which the panelist is invited to present his own views on the central question, followed by a second 5-minute Q&A.

During the last 30 minutes the moderator will open the floor to the audience for dialogue with all the panelists.

Hung-Hsi Wu, University of California, Berkeley

There is at present little effective transition strategy to lead student from arithmetic to algebra. One view is that this should be done gradually in the teaching of fractions. I will give a brief discussion of this point of view.

Robert Moses, the Algebra Project

Paul Sally, University of Chicago
High School Algebra for Middle Grade Students

The Algebra Initiative is a multi-university program in Chicago for preparing teachers to provide instruction in a fully developed, carefully structured eighth grade algebra course.

The Algebra Initiative is conducted by DePaul University, University of Chicago, and the University of Illinois in Chicago. These universities have joined in a collaboration to provide content and strategies for teaching Algebra I at a level that provides a rigorous background to students. Students who take an eighth grade algebra course taught by an Algebra Initiative-trained teacher and pass a citywide proficiency exam may start their high school mathematics program with geometry or advanced algebra.

To qualify to teach this course, a teacher must pass an algebra test based on material from Algebra I and Algebra II. They must also take a yearlong course given by one of the Algebra Initiative universities.

Question 3: What Algebraic understandings are essential for success in beginning collegiate mathematics?

3.1

In this session we discuss what students need from high-school algebra to succeed in entry-level courses that use mathematics. For many students, there is a noticeable gap between what they arrive with and the basic expectations of undergraduate programs. Serious thought about how to address this may include (1) analyzing the ways in which algebra is used in fields outside mathematics, (2) helping students see the purpose for algebraic transformations, or (3) providing better support outside the classroom for students still struggling with algebra. After brief presentations by each panelist, the floor will be open for questions and further discussion.

Deborah Hughes-Hallett
Algebra at College: What Is Its Role?

Exactly how is algebra important in fields outside mathematics? We will make the point that algebra plays a central role in the natural and social sciences. But how is it used there? Are we teaching the kind of algebra that students need? We will characterize the kinds of algebraic concepts and skills that students need to be successful in college.

William McCallum
Creating Mindful Manipulators

Two common prescriptions for students weak in algebra are more drill, or more modeling. Although these can be part of the answer, we also need to encourage students to be mindful manipulators—to look at the structure of algebraic expressions and equations, to make strategic choices about how they will manipulate them, and to understand the purpose of and reasoning behind the manipulations they perform. The algebra curriculum in high school needs to find a middle ground between killing algebraic understanding through an overemphasis on procedure, and veiling it behind an overemphasis on functions. We will give some examples to illustrate what problems and activities might populate this middle ground.

Tom Roby

The University of Connecticut's general education requirements include a "Quantitative (Q) Competency" that all students must meet by taking 2-3 courses flagged with a "Q". The main criterion for being a Q course is "knowledge and use of mathematics and/or statistics at or above the basic algebra level as an integral part of the course". The charge of the Quantitative Learning Center is to support students and teachers in Q courses, just as the University Writing Center supports W (writing intensive) courses. Free peer-tutoring provided by the Q Center finds itself on the front lines supporting students struggling with algebra in their collegiate math and science courses. The volume of students coming in for help has grown dramatically: from 600 visits in Fall'05 to 7000 in Fall'07, providing a useful base from which to study the difficulties students have. In particular, we note how seemingly small changes in instructional design, such as a shift to online homework, can have a significant effect on the pattern and focus of student visits.

3.1a

William McCallum

Glen Stevens
Mining the early mathematics curriculum

The transition from high school to college is difficult for many students. Perhaps the greatest challenge is adjusting one's way of thinking about the nature of mathematics. Algebra, with its focus on algorithms and early techniques of abstraction, presents both opportunities and dangers for students in making this transition. In this presentation, we will discuss "algebraic ways of thinking" and the impact they have on student success in college mathematics courses.

3.1b

James Fey and Dan Chazan
What algebraic understandings do we wish future teachers might gain in college?

In three animations about the solving of linear equations created by the Thought Experiments in Mathematics Teaching (ThEMaT) project, students ask their teachers mathematical questions. We will use these questions, and the mathematical understandings needed to respond to them, to consider the algebraic understandings we wish future teachers might have, and how they might gain such understandings in college.

3.3 Panel: Preparing teachers to teach algebra

Dan Chazan

Students increasingly begin their study of algebra in middle school. In states where there is no middle grades certification, their teachers were prepared to teach elementary school. Challenges involved in preparing such teachers to teach algebra will be addressed by describing the design of a cohort-based masters program for elementary certified middle grades mathematics teachers. This design is the result of the collaboration of two districts with mathematicians and mathematics educators.

Al Cuoco

An examination of the day-to-day work of high school teachers shows that fundamental ideas and results from abstract algebra, linear algebra, and number theory are useful tools for teaching high school algebra and, more generally, for brining coherence and texture to much of the high school curriculum. Furthermore, these ideas and results are useful in lesson planning, task design, and in other “out of classroom'' activities that are essential to the profession. I'll make some suggestions for how undergraduate mathematics courses make explicit connections to these aspects of mathematics teaching.

Hung-Hsi Wu

The main issues surrounding the preparation of teachers to teach algebra cannot be divorced from the general problems of our dysfunctional pre-service professional development. I will give some examples of how we have failed in this endeavor, and discuss some proposals for improvement.