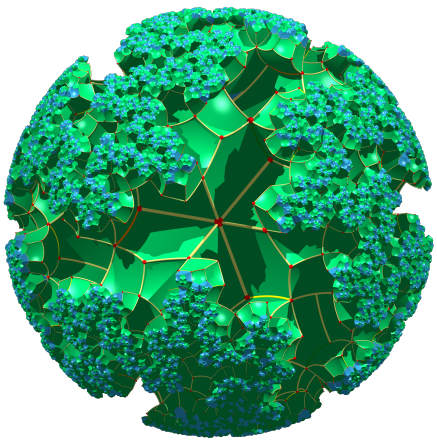
RANDOM AND ARITHMETIC STRUCTURES IN TOPOLOGY

The use of dynamical invariants has long been a staple of geometry and topology, from rigidity theorems, to classification theorems, to the general study of lattices and of the mapping class group. More recently, random structures in topology and notions of probabilistic geometric convergence have played a critical role in testing the robustness of conjectures in the arithmetic setting. The program will focus on invariants in topology, geometry, and the dynamics of group actions linked to random constructions.



Historically, the investigation of arithmetic hyperbolic manifolds blends tools from number theory and geometry. In dimension 3, non-arithmetic examples can easily be constructed as well. Random hyperbolic 3-manifolds are used to analyze the expected geometric and topological properties and uncover the asymptotic behavior of invariants in towers of covers with tools from random walks on groups, group sieving, geometric representation theory, L^2-invariants, homology with twisted coefficients, and global analysis.

The notion of invariant random subgroups (IRS) provides a framework to study invariants of all hyperbolic manifolds, and, more generally, of all locally symmetric manifolds of non-compact type. Indeed, IRSs can be regarded as generalizations both of normal subgroups and of lattices, making this notion particularly rich.

Closely related is the so-called Benjamini–Schramm space which is defined to be the space of all Borel probability measures on the space of isometry classes of pointed proper metric spaces equipped with the Gromov–Hausdorff topology. Benjamini–Schramm convergence provides a more probabilistic notion of geometric convergence; limits capture geometric phenomena that hold almost everywhere.

Measure theory arises more directly in the concept of measure equivalence relation and cost. Entropy for group actions provides a numerical invariant which in turn has an interpretation in terms of topological invariants constructed from the group von Neumann algebra over the group ring. The new concept of sofic entropy can be studied for a large class of groups, the so-called sofic groups, which includes all residually finite and all amenable groups.

A unifying theory or conceptual framework for these approaches towards understanding invariants for the above examples has yet to be fully developed. This semester program will interrogate and deepen the connections between randomness, number theory, and topology by bringing together experts from a wide range of areas of mathematics, whose interest in the general scheme is motivated by concrete questions in geometric topology, number theory, ergodic theory and algebraic groups. Such random questions provide a framework to conjoin central elements in these already well-established fields.