Lars Valerian Ahlfors's *Complex Analysis*
by
Hugo Rossi

"The author has tried to emphasize economy of thought in order to make the reader aware of the intrinsic unity which is so characteristic of the subject." So writes the author in his preface. As I read that the first time I had little conception of what he meant, nor how well he had achieved his goal. "Economy of thought": I was ready for that (so I thought), having come off a year of study of Widder's Advanced Calculus, a work which is hardly gabby. Widder's style is reminiscent of the two column proofs of high school geometry, followed by exercises ranging from trivial to tricky. This turned out to be quite different from Ahlfors's prose style: that of a sequence of essays, followed by problems which did not serve to reinforce the text, but instead, extend and develop it.

Our instructor, Professor Schwartz, warned us that the text was new (having been published two years before) and in "the modern style": we'd have to study the text very carefully and slowly. I took the book home and scanned through the pages. I was surprised that Theorem 1 didn't occur until page 29: what could he be writing about before then? At that, it was introduced as an "application" - of what, I could not detect. I soon learned that this book could not be used as a reference book, and instead was to be read "very carefully and slowly" in the author's chosen order. And indeed, just as the Professor required, one must do all the problems.

So, we worked our way through the text. At first the students' solutions to problems were clumsy and awkward, for we tried to rely on real variable methods. Little by little we began to understand that the interpretation of a complex number as a rotation-dilation of the plane made many of the problems solvable with grace, and soon the problems began to fit in as an essential part of the structure of the subject. The beautiful treatment of fractional linear transformations by means of the cross ratio, and the interpretation as conformal motions of the sphere was a revelation, making us confirmed believers in the efficacy of complex geometric methods.

The Cauchy theorem seemed to me to be a sort of complex fundamental theorem of calculus, and when I suggested this to Professor Schwartz, he said sure: it is a direct application of Green's theorem; just use the Cauchy-Riemann equations. This troubled me: why was Ahlfors suppressing these much easier real-variable methods? Professor Schwartz let the objection lie, and continued through the text. The index formula was a turning point: this was neat and clear. The representation of the inverse of an analytic map as an integral was another milestone: real variable methods would never have suggested this; but here it flowed so inexorably out of the Cauchy integral theory that Ahlfors didn't even feel it deserved its own display as a Theorem. The introduction to homology, the statement and proof of the "general Cauchy theorem" clinched this all for me, and I finally began to get a glimpse of the "intrinsic unity" of which Ahlfors wrote in his preface.

As a graduate student I took a course in Geometric Function Theory from Lennart Carleson. This was my first introduction to truly intense analytic methods. Ahlfors' introduction to the subject gave me the ability to see the geometric intuition driving Carleson's analysis; without it I could not have survived that course

As I look back at my career I see myself as a geometric analyst, even though I have worked in a variety of different subjects. I can now appreciate that it was this little book of Ahlfors that introduced me to geometric methods in analysis, and made those methods the bedrock of my intuition and approach.