- **Powers of the vertex cover ideal of a chordal graph,** *Communication in Algebra*, Volume 39, Issue 10, $2011, 3753 - 3764$.

  As a consequence of Eagon and Reiner's Theorem, we have that a graph is Cohen-Macaulay if and only if its vertex cover ideal has a linear resolution. I use the machinery of the weakly polymatroidals to study the powers of the vertex cover ideals. The results are in the direction to study whether all powers of the vertex cover ideal of a Cohen-Macaulay graph have linear resolutions. In fact we showed that the answer is yes in the case of chordal graphs. I also discuss on a conjecture due to Herzog, Hibi and Ohsugi that all powers of the vertex cover ideal of a chordal graph are componentwise linear.

- **Monomial ideals and toric rings arising from a poset**, with V. Ene, J. Herzog, *European Journal of Combinatorics* $32, 2011, 404 - 421$.

  In this paper we study monomial ideals attached to posets. We introduce generalized Hibi rings and investigate their algebraic and homological properties. The main tools to study these objects are Gröbner basis theory, the concept of sortability due to Sturmfels and the theory of weakly polymatroidal ideals. Motivated by the dual relationship between Hibi ideals and edge ideals of bipartite graphs the new concept, generalized Hibi ideals, is a natural way to extend the result to multipartite graphs. We get an explicit description of the maps in the resolution of these ideals. With this information at hand we can describe the projective dimension of them as the maximal length of antichains in the corresponding poset. Also their toric rings which naturally generalize the classical Hibi rings have been studied. We showed that all toric rings of some subclasses of these ideals are normal Cohen-Macaulay domains.

- **Some algebraic properties of hypergraphs,** with E. Emtander, S. Moradi, *Czechoslovak Mathematical Journal* Vol. 61, No. 3, 2011, 577–607.

  We consider StanleyReisner rings $k[x_1, \dots, x_n]/I(H)$ for the edge ideals associated to some particular classes of hypergraphs. For instance, we consider hypergraphs that are natural generalizations of lines and cycles, and for these two families we compute the Betti numbers. Then we generalize the well-known result that Δ has linear quotients if and only if the Alexander Dual of Δ is shellbale. Indeed we define two concepts d-shellablity and having d-linear quotient, and we show that they are indeed dual concepts. The notion of being connected is very well behaved in the sense that if we choose the coefficients in the associated chain complex from a field k. We generalized the connected notion from graphs to hypergraphs, and we show that if a hypergraph is homologically connected over the field Q, then it is homologically connected over every field, and it does not depend on the characteristic of the field.

- **Shellable cactus graphs**, with D. Kiani, S. Yassemi, *Math. Scand.* $106,\ 2010,\ 161 - 167$.

  We study a family of cactus graphs in order to give a complete characterization for vertex decomposable graphs and sequentially Cohen-Macaulay graphs among them. Motivated by a result of Francisco, Hà and Villarreal, we study the effect of adding whiskers, ears and cycles to a graph. Our theorems give us a criteria to construct more vertex decomposable graphs by making some modification on graphs. Taking advantage of new constructed vertex decomposable graphs it is shown that a cactus graph is vertex decomposable if and only if it is sequentially Cohen-Macaulay.

- **Weakly polymatroidal ideals with applications to vertex cover ideals**, with S. Moradi, *Osaka J. Math.* 47, 2010, 627 – 636.

  We study a generalization of weakly polymatroidal ideals introduced by Hibi and Kokubo. To be more precise we consider monomial ideals not generated in the same degree necessarily. Then using combinatorial method we show that they have linear quotients. As combinatorial and algebraic consequence of our result it follows that each shadow of a weakly polymatroidal ideal is again a weakly polymatroidal ideal. These ideals have another interesting consequence; the Alexander dual of the simplicial complex corresponding to a squarefree weakly polymatroidal ideal is vertex decomposable. This parallels a result by Herzog, Reiner and Welker who show an analogous result for squarefree weakly stable ideals.

- **On the Betti numbers of the edge ideals of some hypergraphs**, with D. Kiani, S. Moradi, *Beiträge zur algebra und geometry,* 51, 2010, 467 – 475.

  We study a generalization of Ferrers hypergraphs which their edge ideals have linear quotients, and so they have minimal linear free resolution. Combining this fact with a result by Herzog and Takayama on mapping cones, we can get their Betti diagram explicitly. This technic also allows us to compute the Betti numbers of complete multipartite hypergraphs.

- **A shellable and sequentially Cohen-Macaulay graphs of the form** $\theta_{n_1,\ldots,n_k}$, with D. Kiani, *Bulletin of the Iranian Mathematical Society,* Vol. 36, No. 2, 2010, 109 – 118.

  We give a complete characterize for sequentially Cohen-Macaulay graphs of this type. Then we show that for these types of graphs the notions of vertex decomposable, shellable and sequentially Cohen-Macaulay are equivalent.

- **The Total Graph and Regular Graph of a Commutative Ring**, with S. Akbari, D. Kiani, S. Moradi, *J. Pure Appl. Algebra* 213, 2009, 2224 – 2228.

  We study the total graph of a commutative ring, denoted by $T(G(R))$ which is a graph with all elements of the ring as its vertices, and two distinct vertices are adjacent if and only if their sum is a zero devisor element of the ring. Beside other results we gave a complete characterization for the finite commutative rings with Hamiltonian total graph and also the rings in which the induced subgraph on the regular elements is Hamiltonian. Then we showed that each commutative Noetherian ring with finite regular elements is indeed a finite ring.

- **Poincar**$\acute{e}$ **series of some hypergraph algebras,** with E. Emtander, R. Fröberg, S. Moradi, to appear in *Mathematica Scandinavica*, arXiv:0901.1534.

  We determine the Poincar$\acute{e}$ series $P_{R_H}(t) = \sum_{i=1}^{\infty} dim_k \mathrm{Tor}_i^{R_H}(k,k)t^i$ for the hypergraphs H generalizing lines, cycles, and stars. We also calculate the graded Betti numbers and the Poincar$\acute{e}$ series of the graph algebra of the wheel graph.

- **Classification of** $M_3(2)$**-graphs,** with S. Akbari, D. Kiani, S. Moradi, to appear in *Ars Combinatoria.*

  A graph is called an $M_r(k)$-graph if it has no k-list assignment to its vertices with exactly r vertex colorings. We characterize all $M_3(2)$-graphs. it is shown that a connected graph is an $M_3(2)$-graph if and only if each block of the graph is a complete graph with at least three vertices.

- **Arithmetical rank of the cyclic and bicyclic graphs,** with M. Barile, D. Kiani, S. Yassemi, to appear in *Journal of Algebra and its Applications*, arXiv:0805.1657.

  Combining some combinatorial methods and homological techniques we study the numerical invariants of the edge ideal of all cyclic and bicyclic graphs. We prove that the arithmetical rank equals the projective dimension of the corresponding quotient ring. Moreover we show that projective dimension of the edge ideals of these graphs does not depend on the characteristic of the ground field.

- **On the associated graphs to a commutative ring,** with Z. Barati, K. Khashyarmanesh, Kh. Nafar, to appear in *Journal of Algebra and its Applications*, preprint available at my homepage, 12 pages.

  We associate a simple graph to an arbitrary commutative ring and its arbitrary multiplicatively closed subset in which all elements of the ring are vertices, and two distinct vertices are adjacent if and only if their sum belong to the chosen subset. Two well-known graphs of this type are the total graph and the unit graph. In this paper, we study some basic properties of these graphs. Moreover, we unify, improve and generalize some results for the total and the unit graphs attaches to the rings.

- **Arithmetical rank of some cacti graphs,** with D. Kiani, to appear in *Algebra Colloquium*, preprint available at my homepage.

  In this paper we compute the projective dimension of the edge ideals of a family of cacti graphs. We show that the arithmetical rank of these ideals equals to their projective dimension. Applying these results we are able to compute the arithmetical rank of some monomial ideals of higher degree. Determining the arithmetical rank of monomial ideals was pioneered by Lyubeznik. We studied the equality between the projective dimension and the arithmetical rank of the edge ideals of these graphs. In order to show the equality we found a lower bound for the projective dimension, then we describe a sequence of elements of the edge ideals which generates this ideal, up to radical to get an upper bound for the projective dimension. With this technique beside getting the projective dimension and the arithmetical rank, we can determine a sequence of elements of the ideal which generates it, up to radical by Schmitt and Vogel's method. We use different methods as Hochster's Formula and Mayer-Vietoris sequence for the reduced homology of simplicial complexes associated to graphs to find explicit formulas for the Betti numbers of some graphs.

- **Determinantal Facet Ideals**, with V. Ene, J. Herzog, T. Hibi, arXiv:1108.3667v1, 14 pages.

  Motivated by the classical case of determinantal ideals we study the ideal generated by an arbitrary set of minors of a generic matrix of indeterminants. The goal of the project is to study the connection between the algebraic properties of these ideals and their associated simplicial complex. We show that the generators of the ideal form a Gröbner basis for the ideal which they generate with respect to the natural lex order, if and only if the simplicial complex is closed. In this case we computed the numerical invariants of the ring associated ring and we proved that S/J_Δ is Cohen–Macaulay. Then applying Sagbi basis criterion we observed that the K-algebra generated by generators of the ideal is Gorenstein. We also discuss when a determinantal facet ideal is a prime ideal. As a main result we find a necessary combinatorial condition on the simplicial complex to have primality of the ideal.

- **Hilbert function of binomial edge ideals,** with L. Sharifan, submitted, 14 pages.

  Our plan is to study the numerical invariants of the binomial edge ideals. The main idea is to sit the associated ring to a graph in a short exact sequence with the property that two other modules in that sequence are known. For this purpose, we use the information given in a paper by Herzog and Hibi concerning the minimal primes of binomial edge ideals to find the minimal primes of some quotient ideals. A crucial point is to give a combinatorial description for the quotient ideal which can be described by the binomial edge ideal of a new graph, and the monomials corresponding to the pathes between two vertices in the graph. Then applying this description together with the known numerical invariants for the closed graphs we are able to compute some numerical invariants like depth and the Hilbert function of the binomial edge ideals whose graphs are not closed.

- **Resolution of unmixed bipartite graphs,** with S. Moradi, submitted arXiv:0901.3015v1.

  We describe the multigraded minimal free resolution of the edge ideal of these graphs. More important is the fact, that the multigraded basis elements of the resolution, can be identified with the Boolean sublattices associated to graphs. Having this identification, it turns out that the multigraded extremal Betti-numbers can be count by the Boolean sublattices corresponding to such graph. Then the depth and regularity of the edge ideal can be expressed in terms of its associated lattice.

- **An algebraic criterion for the choosability of Graphs,** with S. Akbari, D. Kiani, S. Moradi, submitted, 10 pages.

  Applying a result by Alon and Tarsi we study the f-choosability of a graph G for a given function. We provide a criterion for f-choosability of a graph. Using this criterion we get the choice number of some complete multipartite graphs, and we give another proof for a well-known result due to Erdös, Rubin and Taylor concerning the choice number of K_{2*r}.

- **Cohen-Macaulay Cactus graphs**, preprint.

  In this paper all Cohen-Macaulay cactus graphs (i.e., connected graphs in which each edge belongs to at most one cycle) are characterized. Using the combinatorial characterization given for these graphs, I show all powers of the vertex cover ideal of a Cohen-Macaulay cactus have linear resolutions. Then I describe a sufficient combinatorial condition on an arbitrary graph which guarantees that all powers of its vertex cover ideal are componentwise linear.

- **Strongly shellable bipartite graphs,** with D. Kiani, S. Moradi, S. Yassemi, submitted 10 pages.

  We present a characterization of shellable bipartite graphs inspired by a criterion introduced by Herzog and Hibi. Then by identifying bipartite shellable graphs, we explore how adding different configurations of whiskers to a Cohen-Macaulay bipartite graph affects the property of shellablity.