Statement of Research by Brandy S. Wiegers

My current research focus is the development of a dynamic, three-dimensional model of the water movement in the 10 mm primary growth section of plant roots. This project requires a proficiency in numerical analysis, general diffusion models, mathematical biology and plant physiology. My current work allows for a ten-year plan, providing both graduate and undergraduate research problems. In addition to my current work, I have experience with a wide variety of applied mathematical applications including bio-statistical modeling [3], population dynamics, phylogenetics and engineering applications. This broad range of scientific experience allows for quick adaption to new research questions based on the work of collaborators at my future institution.

Current Research: Modeling the Growth-Sustaining Water Needs of a Plant Root

The primary aim of my current research is to understand the water transport process that allows for plant root growth. The root growth process is characterized by the plant cells absorbing water which, in turn, generates pressure on the cellular wall. This pressure forces the cell to grow by inflating it like a water balloon, stretching the cellular wall to the point that the expansion is irreversible. The primary question that plant physiologists have about this process is, where is the water coming from to enable the growth?

There are several root growth water source theories. The **External Source** theory is focused on external water sources, assuming that the only source of water to facilitate the growth process is the surrounding soil with no water being provided from the rest of the plant [1,5]. Wiegers, et. al. propose the **Multiple Source** theory of root growth, which assumes in addition to the external water source there are also internal sources. In other words, we assume that the internal tissue of the plant transports water for growth into the growth zone from the mature tissue higher in the root (see Fig 2(a)) [8].

I have tested the External and Multiple Source theories by examining the *water potential* (ψ). Water potential is a measure of the free energy of plant cell water relative to pure water. More importantly, the gradient of water potential is the driving force for water movement in plants; the water potential quantifies where the water moves within the plant. We will understand the resistance to water movement through the plant by considering the interior *hydraulic conductivity* (K). Combining K with the *relative elemental growth rate* (L), the spatial distribution of growth within the root organ, we can solve for the *water potential* gradient using:

$$L = \nabla \cdot (K \cdot \nabla \psi) \tag{1}$$

The theories can be tested by evaluating (1) using numerical methods to estimate the water potential gradients during root growth and comparing the gradient estimates to experimental observations.

Computational Methods

To facilitate the numerical approach to this problem, I created a body-fitted grid that approximates an average corn root, see Figures 2(a), 1(b) and 1(c). I generated the outer grid surface by averaging the boundary coordinates of corn roots in micrograph photos, and the internal grids were created using a parabolic longitudinal grid combined with a cross-sectional H-grid.

Numerical computation on the body-fitted grids was simplified by the use **generalized coordinates**, a tool that takes any computational grid with components (x,y,z) and relates it to a regular orthogonal grid with components (ξ, η, ζ),

Variable	**Definition**
Relative elemental growth rate (L)	A measure of the spatial distribution of growth within the root organ. Units: hr^{-1}
Hydraulic Conductivity (K)	A measure of the ability of water to move through the plant. K is inversely proportional to the resistance of an individual cell to water influx. Values of K can be quite variable, depending on factors including growth conditions and intensity of water flow. Units: $\text{m}^2\ \text{MPa}^{-1}\ \text{s}^{-1}$
Water potential (ψ)	A measure of the free energy of plant water relative to pure water. The gradient of water potential is the driving force for water movement in plants. Units: MPa

Table 1: Root Growth Variables [6]

see Figure 1(a), [2]. Generalized coordinates allow us to solve (1) in terms of an orthogonal square computational grid by providing a translation that relates the orthogonal grid solution to the original root-shaped computational grid solution. We see the true beauty of the generalized coordinates approach when reviewing a center finite difference approximation of g_ξ (a general function g differentiated with respect to ξ),

$$g_\xi(i,j,k) = \frac{g(i+1,j,k) - g(i-1,j,k)}{2 * \Delta\xi} + O(\Delta\xi^2) \tag{2}$$

ξ represents a variable from a regular, equally spaced computational grid so $\Delta\xi$ is defined equally for all computational points (i,j,k). This is much easier to work with than re-calculating the changing Δx, Δy and Δz in the root-computational grid (see 1(b) and 1(c)). The use of generalized coordinates is especially important for this model because it provides the ability to perform additional grid refinement and other grid changes without the need to re-program the solver or change the equations. This allows for quick improvements of the scientific accuracy of the model.

Generalized coordinates and finite difference approximation (2) are used to convert the partial-differential equation (1) into a linear system of equations represented in matrix form by:

$$[Coeff]\psi = \mathbf{L} \tag{3}$$

The relative elemental growth rate vector ($\mathbf{L}$) and the coefficient matrix ([Coeff]) are calculated with known physiological values. (3) is then solved to find the unknown internal water potential values ($\psi_{i,j,k}$). The results are plotted in three-dimensions to visualize the water potential gradient (see Fig 2(b), Fig 2(c)).

Modeling Results

Results of this model of water potential gradient visualize the water potential and are providing valuable insight into the plant root growth theories. Using the assumptions of the **External Source** theory (see Fig 2(b)), the model calculates a radial gradient of water potential that has not been verified empirically [1, 4, 5]. In contrast, the results of the **Multiple Source** theory assumptions (see Fig 2(c)) are promising when compared to experimental data because of the decreased radial gradient as a result of the addition of internal sources. This success has encouraged further study of the Multiple Source Theory. I am currently extending the model by adding time in order to study non-steady state growth. The result will be a simulation of the water movement in a plant root growth zone during a one-hour period. This simulation will allow for a computer visualization of growth and a more thorough sensitivity analysis of the model.

Future Work

Over the course of my career, I plan further work on this model in order to gain insight into the soil-root relationship. Specifically, I plan to examine the drought root growth condition. The primary root growth zone is the only part of the plant that is still active under drought conditions. So, the water relations within this section of the root is an important area of study that will provide additional modeling problems. Due to the global socio-economic impact of drought, this line of research has many funding opportunities. Through my affiliation with the **Resource Modeling Association**, which has a focus on forestry resources, I know that there is also much interest in collaboration on this type of resource model. In addition, the model is such that it can continue to be developed into a series of smaller research projects to meet the interests of students working with me as there are a number of numerical and theoretical problems available to be studied.

Future Work: Developing a Research Program

Beyond my current research, I have been involved with several other research projects including organizing and teaching with the *Collaborative Learning at the Interface of Mathematics and Biology (CLIMB)* program, organizing and teaching with the *Math Modeling Experience (MME)* and participating in four different undergraduate research projects. These experiences have provided extensive background in:

- Mathematical Biology: Population and Social Dynamics, Bio-fluids, Marine Reserves, Phylogenetics, Cellular Motility, Gene Regulation, Plant morphogensis, active media pattern formation and electrically excitable cells.
- Engineering Topics: Wetlands Treatment Design, Bio-Medical Design, Thermodynamics.

- Medical / Bio-stastics/ Epidemiological Problems, with a current publication in prostrate cancer research [3].

This background prepares me to develop a research program that will evolve to meet the interests of the students and colleagues that I work with.

One important factor in the successful development of my research program is the utilization of student (undergraduate and graduate) research experiences. When I attended **Transforming the Culture: Undergraduate Education and the Multiple Functions of the Research University**, I learned much about the development of undergraduate research programs [7]. This conference reinforced the conclusions I've developed in my work with the **Collaborative Learning at the Interface of Mathematics and Biology** (CLIMB) and **Math Modeling Experience** (MME) programs at UCD. Specifically, the conference singled out the importance of a well-planned student research experience and a need for a focused mentorship process in creating successful future researchers. My work with UC Davis CLIMB and the UC Davis MME students has verified the importance of these program aspects and has provided the skills and experience to make me successful in creating my own programs.

CLIMB: Collaborative Learning at the Interface of Mathematics and Biology
The concept behind CLIMB is simple. In order to create successful mathematical biology collaborations, universities should train undergraduate mathematics and biology students how to collaborate. CLIMB is a year-long program that brings together 7-9 undergraduate students and orchestrates their learning process, group dynamics and mathematical and biological skill set to allow for a collaborative summer research project. As the Fall CLIMB assistant, I have first hand experience with the difficulty and reward of bringing together a group of students with diverse educational and research backgrounds and helping them complete collaborative projects. In this program, I have worked with ten different research projects, learned how to manage undergraduate research collaboration and gained insight into student research process that will influence and provide the necessary experience to successfully coordinate my own student research projects.

MME: Math Modeling Experience
The Math Modeling Experience is a graduate student organized series of courses for high school and undergraduate students that prepares them for the COMAP (Consortium of Mathematics and its Applications) MCM (Mathematical Contest in Modeling) and HiMCM (High School Mathematical Contest in Modeling) competitions. As an instructor for this program, I prepared several 2 hour lesson plans that introduced students to math modeling topics including disease and population growth models. The goal of my lessons was to introduce the students to a wide variety of model techniques while teaching them the tools to develop their own models to address potential competition problems. Our program was very successful; teams of both high school and undergraduate students received a rank of outstanding in the competition. Their solutions set them apart from other teams across the nation.

CLIMB and MME are examples of the type of research program that I would like to develop and be involved in at a new institution. Both programs allow for further integration of research into the teaching process, providing important hands-on research experience for the students that participate. In addition, MME is an undergrad program that encourages collaboration between academic researchers, K-12 educators and industry researchers. This collaboration fits the current needs of California and opens the possibility of grant support. I was the lead author on the successful grant that funds the Math Modeling Experience at UCD, and I plan to use my knowledge from this process to develop and fund similar programs in the future.

Conclusion

My current scientific computational work using numerical methods, linear algebra and partial differential equations to answer questions in plant physiology has already made advances in the field and provides the opportunity for future research projects. In addition, my involvement and exposure to a broad range of projects in mathematical biology and computational mathematics provide the background that will lead to a fully developed research program that will attract external funding to support both my research and the research of my students.

References

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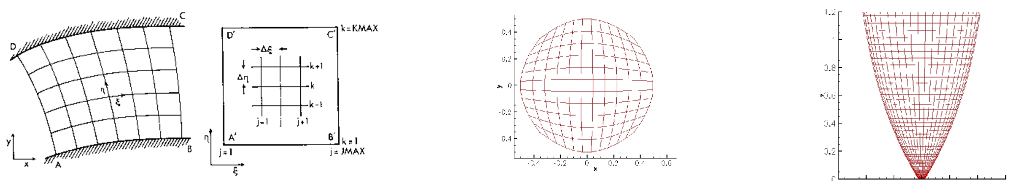
Figures

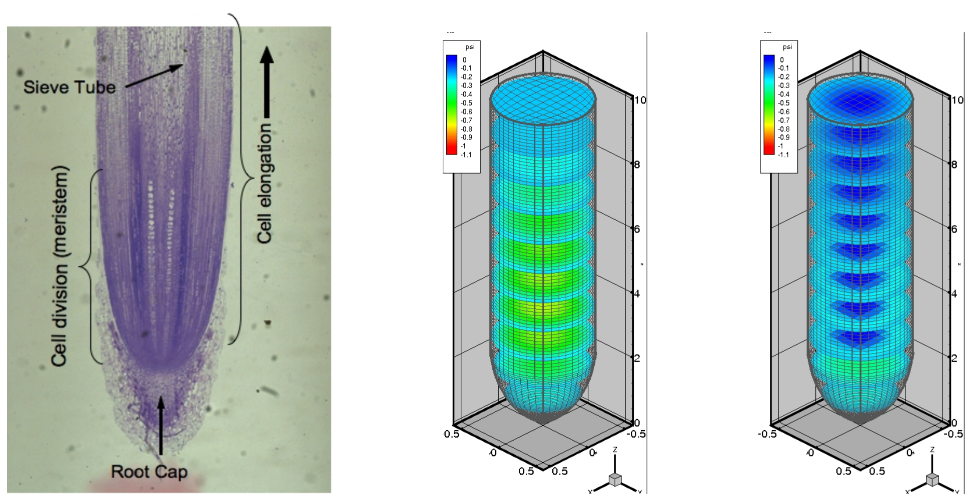


(a) Overview of Generalized Coordinates approach in 2-d, converting the general grid (x,y) into the regular orthogonal (ξ,η) grid. [2]

(b) 2-d view of the H-grid used for the internal radial grid.

(c) Closer view of the parabolic r-z grid used modeling the 2 mm root tip.

Figure 1: Computational Grids and Generalized Coordinates



(a) The primary root growth zone

(b) External Source Model.

(c) Multiple Source Model

Figure 2: Water Potential Distributions in a three-dimensional plant root tip. (2(a))The primary root growth zone of a corn root. (2(b), 2(c)) Radial cross-sections of the model results, embedded in the computational grid. The coloring is contoured, relative to the most negative water potential value ψ_{min}, with the gradient beginning at $\psi = 0$ (blue) and decreasing to ψ_{min} (red). (2(b)) External Source model has a longitudinal and radial gradient. (2(c))Multiple Source model has a longitudinal gradient and a much smaller radial gradient (relative to 2(b)).