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Corrections to

Commutative Algebra with a View Towards Algebraic Geometry

by

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Introduction

p. 6 line 2* and p. 8, line 2* Macdonald is misspelled (the D should be d).

p. 7 line 17: replace " is" (at end of line) by "are"

p. 7 line 3*: replace 1.11 by 1.9

Chapter 0.

p. 11 line 12*: Replace this line by:

A **ring** is an abelian group R with an operation $(a, b) \mapsto ab$ called *multiplication*

p. 11 lines 6*-4*: Replace the sentence beginning "Nearly. . . by " In this book we use the word ring to denote a commutative ring, with a very few exceptions that will be explicitly noted."

p. 11 line 1*: after "in which" insert $1 \neq 0$ and

p. 12 line 8: replace R by I

p. 15 line 21: change "$I = (i), J = (ij)$, and i is a nonzerodivisor" to "$I = (ij), J = (j)$, and j is a nonzerodivisor"

p. 15 line 17*: Add to this paragraph the sentence: "For example, we say that a nonzero element $r \in R$ is a **nonzerodivisor on** M if $(0 :_M r) = 0$; that is, if r annihilates no nonzero element of M."

p. 16 line 11: "Corollary 4.5" should be "Corollary 4.4"

Chapter 1.

p. 16 line 2*: “generated” should be “generate”

p. 22

l ine 3*: change “unfortunately” to “but”

l ine 2*: delete the words “after all”

l ine 1*: replace “spawned.” with “spawned, including a large chunk of commutative algebra. The amazing recent proof of Fermat’s last theorem by Wiles and Taylor continued this tradition: a small but significant step in the proof involves a novel argument about Gorenstein rings.)

p. 23 line 5: after [1881] insert (see also Edwards [1990])

p. 24 line 14*: change [1992] to [1993]

p. 24 line 3*: replace R by k

p. 26 line 3 of the footnote: after “celebrated” insert (see for example Browder [1976] or Kaplansky [1977])

p. 26 last line of the footnote: change [1986] to [1990]

p. 27 line 19: before the word “collection” insert “nonempty”

p. 28 line 15*: before the word “collection” insert “nonempty”

p. 29 lines 20*-18*: Change the long sentence beginning “If G...” to:

If G is $SL_n(k)$ or $GL_n(k)$ we suppose further that the matrices $g \in G$ act on k^r as matrices whose entries are rational functions of the entries of g; such a representation of G is called *rational* .

p.30 line 7: “for j” should be “for $j >> 0$”.

p.31 line 11: before the period, insert “is a homogeneous polynomial”.

p.31 line 20. Replace the last sentence of the proof by: Thus any homogeneous element of R is in R'. But if $f \in R$ is any element, then applying ϕ to each homogeneous component of f we see that f is a sum of homogeneous elements of R, so we are done.

p.32 line 10: “called an affine” should be “called affine”

p.32 line 21*. insert “nonempty” at the beginning of the line. Near the end of the line, change “to two” to “of two nonempty”.

p.33 line 4*. Change $\mathbf{A}^n_k$ to $\mathbf{A}^n(k)$

p. 37 line 1*. Change fg to $f \circ g$

p. 41 line 15*. Before “algebraic” insert “projective”

p. 42

line 13* Change x_1 to x_0

line 12* Change $r-1$ to r

p.43 line 12: change "integer s" to "integer $s \geq 0$"

p.43 line 14, 15: change " $H(s)$. For all s we have" to "$H(s)$, and"

p.46 line 2: the last "$s+d$" should be $s+d+1$

p.47 line 6*: change ϕ to π

p.49 line 11: change "imples" to "implies"

p.49 line 16: change "$Z(X)$" to "$I(X)$"

p.55 line 9: before the period at the end of the line insert "and the empty set".

Chapter 2.

p. 58 line 10*. Change $\mathbf{A}_k^r$ to $\mathbf{A}^r(k)$

p. 61 line 4*. change "S is a domain" to "S/I is a domain"

p. 63 line 3. change Hom(to $\mathrm{Hom}_\mathrm{R}($

p. 63 line 4. change Hom(to $\mathrm{Hom}_\mathrm{R}($

p. 63 line 21. change M/A to B/A

p. 73 line 19: "then clearly $M \supset M_i$." should be "the claim follows since $M_i = 0$."

p. 73 line 24: change $M' \cap M_i = M' \cap M_i$ to $M' \cap M_{i+1} = M' \cap M_i$

p. 79

line 5 of problem 2.3 Change "sums" to "inclusions".

line 9 of problem 2.3. Delete the first superscript ∞, and delete from "where" to the end of the sentence. Thus sentence should end: "have $R \cap JR[U^{-1}] = \sum_{f \in U}(J : f)$.

p.82 line 13: replace "if x" by "if $x \in R$

p.85 Exercise 2.23. Add a * to the exercise number (indicating that there is a hint, as added below).

p.85 line 7*: delete the *

Chapter 3.

p.90 line 3 The subscript u in the display should be a subscript U

p. 90 After line 8 insert the following paragraph, set off by smallskips.

Like maximal ideals, primes minimal over a given ideal I exist in *any* ring. To see this, note first that if a set of prime ideals in a ring R is totally ordered by inclusion, then the intersection of these primes is again prime. By Zorn's Lemma we may find a maximal totally ordered subset of the primes containing I, and the intersection of the primes in this subset is necessarily minimal over I. By Theorem 3.1a, the set of primes minimal over I is finite if R is Noetherian. This result generalizes the statement that a nonzero polynomial in one variable can have only finitely many roots.

p.90 lines 19-22. Replace these lines (Cor. 3.2 and the sentence before) by:

Theorem 3.1 yields a surprising dichotomy:

Corollary 3.2. *Let R be a Noetherian ring, let M be a finitely generated, nonzero R-module, and let I be an ideal of R. Either I contains a nonzerodivisor on M or I annihilates an element of M.*

p.91 line 8*: Change "an R-module" to " a nonzero R-module"

p.95 line 15: Change "some ideal I" to "some ideal $I \neq 0$"

p.95 line 7*: Change "primary submodules" to "finitely many primary submodules"

p.96 line 3: Change "of M'." to "of 0 in M'."

p.96 before line 10: After the statement of the theorem, and before the proof, add the following paragraph:

"The language in part c is often stretched, and the module M_i is referred to as the P_i-primary component of M in the given decomposition (it may depend on the decomposition).

p.96 line 14: Change "irreducible submodules" to "finitely many irreducible submodules"

p.100 line 20: change "closed" to "intersections of Ass M with the open"

p.101 line 16*: P should be P_i

p.111 line 6: replace [in press] by [1995]

p.112 line 10*: before "show" insert: "and given a minimal primary decomposition $0 = \cap M_i$ as in Theorem 3.10,"

p.112 line 9*: replace "primary components of 0 in M" by "M_i"

Chapter 4.

p.118 line 2: Replace "basis" by "free basis"

p.119 line 15: Replace "$\mathbf{Q}$" by "$\mathbf{Q}(\sqrt{5})$"

p.121 line 3: Replace (x) by (t)

p.121 line 20: change p to P

p.123 line 5*: "Proposition 4.5" should be "Corollary 4.5"

p.130 line 13: after "that of R" add "(in the sense that every element of the big field is the root of a nontrivial polynomial with coefficients in the small field)"

p.131 line 21: change "3.10" to "13.10"

p.134 line 11: change the subscript $_n$ to $_r$

p.134 line 4*: Delete from the sentence "By Theorem..." to the end of the page. Replace this text by:

By Theorem 4.19 the ring $S := k[x_1, \ldots, x_n]$ is a Jacobson ring, so every prime ideal of S, and in particular every prime ideal that contains I, is an intersection of maximal ideals. Thus $I(Z(I))$ is equal to the intersection of all the prime ideals containing I. By Corollary 2.12, this is $\mathrm{rad}(I)$, proving the formula.

The equality $Z(I(X)) = X$ follows directly from the definition of an algebraic set. The formula just proved shows that if I is a radical ideal then $I(Z(I)) = I$. Thus the functions Z and I are inverse bijections between algebraic sets and radical ideals as claimed. ∎

Chapter 5.

p.146 line 4: change "large i" to "large n"

p.146 line 13: change "later in this chapter" to "after Proposition 5.3".

p.148 line 4: Change "an R-module" to "a finitely-generated R-module"

p.148 line 30 = line 8*: the g_j at the end of the line should be $g_j t$

p.149 line 18: change $B_{\mathcal{I}}$ to B_I

Chapter 6.

p.156 line 16: change "maximal" to "prime"

p.156 line 19*: delete ", $U \cong \phi(U)$"

p.157 line 2: change first character of the line, "X", to "x"

p.157 line 12: Append "We have chosen cases where (most of) the fibers are finite sets."

p.158 line 13*: replace "in not flat" by "is not flat"

p.158 line 7*: X should be x

p.158 line 5*: replace "$= R[x]$" (at the end ofthe line) by "$= k[x]$

p.159 line 13*: Delete the sentence beginning "So that the reader may judge the merits..."

p.159 line 8*-5*: Replace these four lines by

1. If M and N are R-modules, and $\ldots \to F_{i+1} \to F_i \to F_{i-1} \to \ldots \to F_0 \to M \to 0$ is a free resolutios of M as an R-module, then $\mathrm{Tor}_i^R(M, N)$ is the homology at $F_i \otimes N$ of the complex $F_{i+1} \otimes N \to F_i \otimes N \to F_{i-1} \otimes N$; that is, it is the kernel of $F_i \otimes N \to F_{i-1} \otimes N$ modulo the image of $F_{i+1} \otimes N \to F_i \otimes N$.

p. 162 The elementary "proof" offered is garbage — the "it follows" just in the middle of the page is too optimistic. Therefore:

Delete the last 2 lines of page 161 and all but the last 3 lines of p. 162.

p. 163 line 1*: in the display, before "*for all i*" insert " *in M* "

p. 165 line 9*: change "m_j" at the beginning of the line to m_i

p. 166 line 19: change "$1 + st$" to "$1 - ts$"

p. 172 , Exercise 6.5. Change each P (four occurences) to $\mathfrak{m}$, and each Q (two occurences) to $\mathfrak{n}$

p. 173 , line 2*. delete "should be $((x) \cap (x, t)^2)$"

p. 175 , lines 5,6. Replace the sentence beginning "We may think of" by For each prime ideal $P \subset R_0$ with residue field $\kappa(P) = K(R_0/P)$ we have a graded module $\kappa(P) \otimes M$ over the ring $\kappa(P) \otimes R$; Thus M gives rise to a family of graded modules, parametrized by $\mathrm{Spec}(R_0)$.

p. 175 , line 12: replace "Hartshome" by "Hartshorne"

Chapter 7.

p. 180 fig 7.1: in the figure, replace "$k[x, y]_{(x,y+1)}$" by "$k[x, y]$"

p. 181 line 19: replace "$\hat{m}$" with "$\hat{\mathfrak{m}}$"

p. 181 line 20: replace "$\hat{m}_1$" with "$\hat{\mathfrak{m}}_1$"

p. 181 line 24:

replace "$\hat{m}$" with "$\hat{\mathfrak{m}}$"

replace "$\hat{R}/\hat{\mathfrak{m}}\hat{R}_{\hat{\mathfrak{m}}}$" by "$\hat{R}_{\hat{\mathfrak{m}}}/\hat{\mathfrak{m}}\hat{R}_{\hat{\mathfrak{m}}}$"

p. 187 para. 3 line 3: replace (f,g) by (g,h).

p. 188 para. 5 last line: the first e_n should have a bar over it like the others.

p. 188 para. 7: replace "by" with "be".

p. 192 line 13: replace "$\hat{\mathfrak{m}}_n \subset (\hat{\mathfrak{m}}_1)^n$" by "$(\hat{\mathfrak{m}}_1)^n \subset \hat{\mathfrak{m}}_n$"

p. 192 line 18: replace "$/\mathfrak{m}_n$" by "$/\hat{\mathfrak{m}}_n$"

p. 194 line 7* and 6*: after the second "Noetherian" insert ''and $\mathfrak{m}/\mathfrak{m}^2$ is a finitely generated $R/\mathfrak{m}$-module." Replace the following sentence, " The ring. . . '' up to is Noetherian." by : "The ring $\text{gr}_{\mathfrak{m}} R$ is generated as an $R/\mathfrak{m}$-algebra by any set of generators for the module $\mathfrak{m}/\mathfrak{m}^2$. Thus by the Hilbert Basis Theorem (Theorem 1.2) $\text{gr}_{\mathfrak{m}} R$ is Noetherian."

Chapter 8.

p.214 line 15*: change [1935] to [1985]

Chapter 9.

p.226 line 5*: replace "$M =$" by "$(M \cup L) =$"

p.226 line 2*:replace "M" by "$M \cup L$"

p. 228 Replace Exercise 9.1 with the following:

a) Let R be a Noetherian local ring. If the maximal ideal of R is principal, show that every ideal of R is principal, and any nonzero ideal of R is a power of the maximal ideal.

b) Deduce from part a) that if each maximal ideal of a Noetherian ring is principal, then the ring has dimension ≤ 1.

p. 228 18*. Replace "is monic in y" by "is a scalar times a monic polynomial in y"

p. 229 line 2*. replace "every element" by "every nonzero element"

Chapter 10.

p. 234 line 12*: In the section title, replace "Parameter Ideals" by "Ideals of Finite Colength"

p. 235 line 3 of text: replace **parameter ideal for R** by **ideal of finite colength**

p. 235 line 13 of text: replace " a parameter ideal for" by "an ideal of finite colength on"

p. 235 line 11*, line 10* and line 7* (three occurences): replace "is a parameter ideal for" with "has finite colength on"

p. 235 line 5*: replace "a parameter ideal for" with "an ideal of finite colength on"

p. 237 Theorem 10.10: replace "map of local rings" by "map of local rings sending $\mathfrak{m}$ into $\mathfrak{n}$"

p. 246 Add the following exercise:

Exercise 10.14: Let S be a Noetherian ring of dimensionsion d, and let I be a radical ideal of S, that is, $I = \mathrm{rad}(I)$. We consider the problem of finding the smallest number of elements $f_1, \ldots, f_e$ that *generate I up to radical* in the sense that $\mathrm{rad}(f_1, \ldots, f_e) = I$. In the case where $S = k[x_1, \ldots, x_d]$, and k is an algebraically closed field, the ideal I corresponds to an algebraic set X, and the problem is equivalent, by the Nullstellensatz, to the problem of determining the minimal number of hypersurfaces that intersect precisely in X. (In the non-algebraically closed case, the problems are quite different; show that every algebraic set in $\mathbf{R}^d$ is a hypersurface, that is, may be defined by a single equation.)

a. Show that if codim $I = c$ then I cannot be generated up to radical by fewer than c elements. Thus there are ideals in S which cannot be generated up to radical by fewer than d elements.

b. Show that I is generated up to radical by at most $d + 1$ elements, as follows. If I is contained in all the minimal primes of S, then I is nilpotent, so the empty set generates I up to radical. Otherwise, choose as first generator f_1 an element in I but not in any of the minimal primes of S that do not contain I. If codim $I > 0$, factor out (f_1) and do induction on the maximum of the dimensions of minimal primes of S that do not contain I.

c. Suppose that S can be written as a polynomial ring in one variable over a smaller ring, say $S = R[x]$. Show by induction on $d - 1 = \dim R$ that I can be generated up to radical by just d elements, perhaps using the following outline. Conclude that every algebraic set in affine d-space is the intersection of d hypersurfaces. The corresponding theorem is also true in projective space, and can be proved by a modification of the argument given below (see Eisenbud–Evans [1973] or Kunz [1985, Ch. 5] for an account).[1]

[1] These results have a controversial history. Part b was first proved by Kronecker [1881], using difficult arguments from elimination theory; the argument suggested here

i. Show that it suffices to treat the case where S (equivalently R) is reduced. Note that if R is reduced and of dimension 0 then R is a product of fields. Now do the case $d = 1$.

ii. Assuming R is reduced, let U be the set of nonzerodivisors of R. Show $R[U^{-1}]$ is a product of fields, and that you can choose $f_1 \in I$ so that $f_1 R[U^{-1}] = IR[U^{-1}]$.

iii. Show that there is an element $r \in R$ not in any minimal prime of R such that $rI \subset (f_1)$. Factor out r and use the induction hypothesis to find $d-1$ elements that generate $I(R/(r)[x])$ up to radical. Lift these elements to elements $f_2, \ldots, f_d \in I$. Show that together with f_1 these generate I up to radical.

Chapter 11.

p. 258 line 10*-7*. Replace the two sentences "This is an uncountable ... . Thus Pic(R).. ." by "This is an uncountable divisible group. An easy argument show that if R is the coordinate ring of an affine open subset of such a curve then Pic R is a quotient of this torus by a finitely generated subgroup which maps onto **Z**. Thus (still assuming $g > 0$) Pic R is an uncountable divisible group.

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is due to van der Waerden, about 1941. In 1891 Kurt Vahlen announced an example (a rational quintic curve in projective 3-space) that he claimed was not the intersection of 3 hypersurfaces. The subsequent history was once explained to me by Alfred Brauer: According to him, Vahlen abandoned mathematics and became, after the first world war, a University Rektor (President), and a prominent Nazi. Perhaps because of this, Oscar Perron was moved to re-examine the example, Vahlen's only significant mathematical contribution, and showed that it was wrong! By hard computation Perron [1941] exhibited 3 hypersurfaces that intersect in Vahlen's curve. Later, Kneser [1960] showed that any algebraic set in projective 3-space is the intersection of 3 hypersurfaces. The proof for affine d-space outlined below is from Storch [1972] and Eisenbud–Evans [1973]. The question of whether curves in projective 3-space can be expressed as the interesection of just two hypersurfaces remains tantalizingly open, even in simple concrete cases; see the discussion at the end of Ch. 15.

Chapter 12.

p. 271 line 13*: bad line break.

p. 272 line 6:replace “parameter ideal $\mathfrak{q}$ for M” by “ideal of finite colength on M”

p. 272 line 7-8: replace “parameter ideal $\mathfrak{q}$ of M” by “ideal $\mathfrak{q}$ of finite colength on M”

p. 272 line 20-21: replace “a parameter ideal for M” by “an ideal of finite colength on M”

p. 275 line 14*-13*: replace “parameter ideals for” by “ideals of finite colength on”

p. 275 line 7*: replace “a parameter ideal” by “an ideal of finite colength”

p. 275 line 3*: replace “parameter ideal” by ”ideal of finite colength”

p. 276 line 7: replace “parameter ideal” by ”ideal of finite colength”

p. 276 line 6*: replace “a parameter ideal for ” by ”an ideal of finite colength on”

p. 277 line 5: delete “the” (third word in the line).

p. 277 line 13: replace “a parameter ideal for ” by ”an ideal of finite colength on”

p. 278 line 2-3: replace “a parameter ideal of ” by ”an ideal of finite colength on”

p. 279 line 17*: replace “any parameter ideal for ” by ”any ideal of finite colength on”

p. 279 line 2*: replace “a parameter ideal for ” by ”an ideal of finite colength on”

Chapter 13.

p. 283 line 8*: change $d_m > 0$ to $d_m \geq 0$

p. 283 line 6*: change $x_{d_{j+1}}$ to x_{d_j+1}

Chapter 14.

Chapter 15.

p. 317 line 1: replace "Restkalssen-" with "Restklassen-"

p. 319 line 11: Delete the ' after "Sturmfels"

p. 320 line 20*: after "largest monomial in F" insert ", in the sense of divisibility,"

p. 322 , 1st line of paragraph before last display:

> "We may now assume that $\sigma = \sum a_v n_v^\epsilon (\ker \phi)_n$" should be "We may now assume that $\sigma = \sum a_v n_v \epsilon_v \in (\ker \phi)_n$"

p. 322-323 Delete the last two lines of p. 322 and the first 12 lines of p. 323 (Lemma 15.1 bis and its proof).

p. 325 line 8*: change m_i to $u_i m_i$

p. 332 line 15: replace $(g_1, \ldots, g_t) \in F$ by $(g_1, \ldots, g_t) \subset F$

p. 332 line 14* to p. 333, line 4: replace these paragraphs by

choose an element

$$f = \sum_u f_u \epsilon_u \qquad \text{with} \qquad \text{in}(\phi(f)) \notin (\text{in}(g_1), \ldots, \text{in}(g_t)).$$

Let m be the maximal monomial that occurs among the terms $\text{in}(f_u g_u)$. We may assume that the expression for f is chosen so that m is minimal, and so that the number of times m occurs among the $\text{in}(f_u g_u)$ is also minimal. Since $\text{in}(\phi(f))$ is not divisible by an $\text{in}(g_i)$, the terms of the $f_u g_u$ that involve the monomial m must cancel; in particular, there must be at least two such terms, and renumbering the g_u we may assume for simplicity that $\text{in}(f_1 g_1)$ and $\text{in}(f_2 g_2)$ are among them. We may write these terms in the form $n_1 m_1$ and $n_2 m_2$, where n_i is a term of f_i.

Since $n_1 m_1$ and $n_2 m_2$ differ by only a scalar, n_1 is divisible by $m_2/\text{GCD}(m_1, m_2)$, and thus there is a term $n \in S$ such that $n m_{2,1} = n_1$. Consider the element

$$f' = f - n(\sigma_{1,2} - \sum f_u^{1,2} \epsilon_u),$$

and write it in the form $f' = \sum f'_u \epsilon_u$. By our hypothesis $h_{ij} = 0$ we see that $\phi(f') = \phi(f)$. The terms of $\phi(n f_u^{ij} \epsilon_u)$ are all $< m$. The term $n_1 \epsilon_1$ in f cancels with the term $n m_{2,1} \epsilon_1$ of $n\sigma_{1,2}$, so this term is missing from f'. The other term $n m_{1,2} \epsilon_2$ of $n\sigma_{1,2}$ combines with the term $n_2 \epsilon_2$ of f so that the number of occurences of m among the $\text{in}(f'_u g_u)$ is strictly less than among the $\text{in}(f_u g_u)$. This contradicts the minimality property of f, so $g_1, \ldots, g_t$ is a Gröbner basis after all. ∎

p 333 line 15*ff: Delete from the paragraph that begins "There is a fairly sharp..." through the end of the page.

p 334 line 1: replace “This estimate is so large as to suggest” by “Worst-case analysis of Gröbner bases shows that the degrees of the elements in a Gröbner basis may be extremely large, suggesting”

p. 337 Footnote: replace “was a postdoctoral student” by “was a young visitor”

p. 339 line 6: delete the word “graded”

p. 339 line 8*-7*: replace “inequality” by “inclusion”

p. 340 line 11*: replace “monomials” by “terms”

p. 342 line 5*: The first $>$ should be $>_\lambda$

p. 347 line 8*: replace “term of the equation is y^2” by “term is $-y^2$”

p. 351 line 2*: Replace the $(1+\delta)$ with $(1+\gamma)$

p. 352 line 6: replace δ_n by δ_r

p. 353 display on line 13*: After $\binom{t}{s}$ insert c^s

p. 359 line 19*: replace p' by g' and p by g

p. 360 line 8: replace I' by I. Replace “the $\text{in}(g_i') = \text{in}(g_i)$.” by “the $\text{in}(g_i)$.

p. 372 Exercise 15.39, replace “I' is any” by “I' is a homogeneous”.

Chapter 16.

p. 392 5th line from the bottom: The direct sum symbol $\oplus_R$ should be $\otimes_R$.

p. 393 lines 9 and 12: the two occurences of $\otimes_R$ should both be $\otimes_{S_i}$

p. 394 Corollary 16.6, displayed equation: add two pairs of parentheses, to make it

$$\Omega_{T/R} \cong (T \otimes_S \Omega_{S/R}) \oplus (\oplus_i T dx_i).$$

p. 394 Corollary 16.6, line 2 of the proof: add two pairs of parentheses to the expression on the right hand side of the equation, to make it $(T \otimes_S \Omega_{S/R}) \oplus (T \otimes_{T'} \Omega_{T'/R})$

p. 394 Corollary 16.6, line 3 of the proof: add a pair of parentheses to make $= T \otimes_{T'} \oplus_i T' dx_i =$ into $= T \otimes_{T'} (\oplus_i T' dx_i) =$

p. 394 Theorem 16.8, line 2: the roman “B” should be a caligraphic B, as on the first line.

p. 394 Theorem 16.8, line 3: the map at the end should be $\phi : S' \to S$.

p. 402 Theorem 16.19, line 3: Change $K(R/P)$ to $K(S/P)$.

Part III

p. 417 line 11*: [1858] should be [1848]

Chapter 17.

p.424 line 4: insert the missing rightarrow in the display between $\wedge^n N$ and $\wedge^{n+1} N$; should be

$$\cdots \to \wedge^{n-1} N \to \wedge^n N \to \wedge^{n+1} N \to 0.$$

p. 439 line 8. after " is a subset" insert " of length s of"

p.439 line 17: $\sum_J c_{I,J}$ should be $\sum_J c_{I,J} e_J$

Chapter 18.

p. 466 second line of Exc. 18.11: Insert the word "primitive" before "polynomial".

p. 477 In Corollary 19.11, delete the phrase ", and let F ... monomial order".

Change the period at the end of the display to a comma, and. after the display add the phrase "where the generic initial ideal is taken with respect to reverse lexicographic order."

p. 482 line 11: after the word "respectively", and before the semicolon, insert: (exept that in the case of the Cayley numbers it is non-associative)

Chapter 19.

p. 487 Exercise 19.18. Delete the second sentence of the exercise, beginnning "Suppose the degrees...".

Chapter 20.

p. 494 First word of the second line of the Proof of Prop. 20.6: "localiztion" should be "localization" is misspelled in the second line of the proof of Prop. 20.6.

p. 506 line 4*: M should be M'

Chapter 21.

p. 522 line 10 replace "A, P" by "(A, P)"

p. 522 line 3*: replace ω_A by $D(A)$

p. 541 Theorem 21.23: Delete the first occurence of "$J = (0 :_A I)$".

p.550 Exercise 21.15 should have the hypothesis that $dim(R) = c$, or more generally that R/J is Cohen-Macaulay.

p. 551 Exercise 21.19: Add a ")" at the end of the first line.

Appendix A1.

Appendix A2.

p. 576 end of first paragraph: There should be a space between "2" and "(bi)".

p. 577 : The right margin is ragged.

p. 596 Fig. A2.7: in the middle of the middle row, the $\wedge^q - gG$ should be $\wedge^{q-g}G$. Also, there is an extra = floating below the symbol $\sum_{p+q-k}$ which should be deleted.

p. 597 Lemma A2.11: The numerals (and perhaps the "A"? in "Theorem A2.10" should be typeset in roman. (Compare with similar situations in Lemma A2.5 and Proposition A3.17.)

Appendix A3.

p. 614 Footnote: Add "(In later editions of the book Lang takes a more moderate position; see Lang [1993].)"

p. 620 lines 4 and 6: The small italic "o" in the statement of Proposition A3.5 and in the first line of the proof should be the number "0" (zero).

p. 629 line 12 (last line before the second display): two pairs of () should be removed, and one (added, so that the final equation should be $\alpha x - \beta x = \partial(h(x)) - h(\partial(x))$.

p. 629 The caption of Figure A3.2 needs to be fixed: In the third line of the caption δ's should be ∂'s. The last line of the caption should be $\alpha x - \beta x = \partial(h(x)) - h(\partial(x))$

p. 630 line 9*: "A3.," should be "A3.13,"

p. 632 line 1*: in the display replace F'' by $F'' \to 0$

p. 650 line 9*: "differential of F" should be "differential of $F[-1]$"

p. 678 line 8: Insert a comma after "Chapter 1]" and after "[1986]," add "Gelfand-Manin [1989],"

p. 679 line 8: H_i should be H_n

p. 681 lines 16 AND 18: "$\to KF$" should be "$\to P \circ KF$"

Appendix A4.

p.683 line 10*: replace "zeroeth" by ' "0^{th}"

p.687 line 8: after $Q \neq P$ insert before the period: "and that all the minimal primes of R have the same dimension"

p.687 line 9*-6*: replace these lines (first four lines of ex A4.3 by

Exercise A4.3: Let (R, P) be a Noetherian local ring, and suppose that $x_1, \ldots, x_n$ generate P. For each $i \geq 0$ there is a natural map $H^i(K(x_1, \ldots, x_n)) \to \mathrm{H}^i_P(R)$ from the cohomology of the Koszul complex to the local cohomology. For $i = 0$ this map is injective, but in general it is neither injective nor surjective. We say that R is **Buchsbaum** if these maps are all surjective for $i < \dim R$. It turns out that this somewhat unappetizing definition leads to a rich and surprizing theory, initiated in Vogel [1973]; the definitive exposition is given in Stückrad and Vogel [1986]. Show

p.687 line 5*: before the word "projective" insert "locally Cohen-Macaulay"

Appendix A5.

p.690 lines 2-4: Replace the sentence beginning "All the methods..." by: "There are difficult foundational issues, but they do not seem to threaten the use of categorical language in common situations. For the best-known way out, see Grothendieck [1972, Ch. I]; for a review, see Feferman [1969]. We shall take a naive approach, and simply ignore the problem."

p.691 lines 8: Replace $M \to N$ by $M \to M$

p.692 line 3: replace $A \to A'$ by $\phi: \ A \to A'$ and replace $B \to B'$ by $\psi: \ B \to B'$

p.692 diagram, symbols next to the left hand vertical arrow: replace $G(\phi)$ by $G(\psi)$.

p.692 diagram, bottom row left hand side: replace A' by A

Appendix A6.

Appendix A7.

p. 710 line 1: replace "algebra" with "algebraic".

p. 710 line 16: replace "Theory" with "theory"

Hints and Solutions for Selected Exercises.

p.714 Before the hint for Exercise 2.27, add a hint for exercise 2.23, as follows:

Exercise 2.23: Check and use the fact that if an ideal I is contained in a principal ideal (a) then $I = a(I : a)$ to show that if every ideal containing an ideal I is principal, then I is either principal or prime.

p. 730 line 4 "J. Sally" should be "S. Wiegand".

p. 733 line 3 of hint for 18.11: after "because" insert "f is primitive we must have $d, e \geq 1$, and because"

p. 741 line 4 of hint for A3.13: in the displayed diagram the subscript "F" should be "F"

References.

p. 747 The reference to Bass appears out of order. It should come before the Bayer references.

p. 747 Insert the reference:

Browder, F. (1976). *Mathematical developments arising from Hilbert's problems.* Proc. of Symp. in Pure Math. 28. Amer. Math. Soc., Providence, RI.

p. 749 before line 4* Insert the reference:

Edwards, H. M. (1990). *Divisor Theory.* Birkhäuser, Boston.

p. 750 Insert new reference: Eisenbud, D. and Evans, E. G. (1973). Every algebraic set in n-space is the intersection of n hypersurfaces. Inv. Math. 19, pp. 107-112.

p. 750 line 11: add "75, pp. 339–352."

p. 750 line 16: add "To be published in revised form by Springer-Verlag, under the title *Why Schemes?*."

p. 750 line 17*: add "109, pp. 168–187."

p. 750 line 13*: Change "1994" to "1996", and add at the end of the line "Duke J. Math. (in press)."

p. 750 before line 7*: Add the reference:

F eferman, S. (1969). Set-Theoretical Foundations of Category Theory. in *Reports of the Midwest Category Theory Seminar*, Ed. S. Maclane, Springer Lect. Notes in Math. 106, pp. 201–247, Springer-Verlag, New York.

p. 752 line 19: change 593 to 523

p. 753 The paper of Heinzer, Ratliffe, and Shah has now appeared; the reference is Houston J. Math, 21, 29-52 (1995)

p. 754 After the references to David Hilbert's papers, insert (Hilbert's papers above are noe available in an English translation in **Hilbert's Invariant Theory Papers**, transl. M. Ackermann. Lie Groups: History, Frontiers, and Applications Volume VIII, Math Sci Press, Boston Mass, 1978.)

p. 758 line 5*, reference to Peskine's book: remove the accent from the word "Algebraic".

p. 755 Insert new reference:

Kaplansky, I. (1977). Hilbert's problems, Lecture Notes in Mathematics, Univ. Chicago, Chicago, IL.

p. 755 Insert new reference: Kneser, M. (1960). Über die Darstellung algebraisher Raumkurven als Durchschnitte von Flächen. Arch. Math. 11, pp.157-158.

p. 758 Insert new reference: Perron, O. (1941). Über das Vahlensche Beispiel zu einem Satz von Kronecker. Math. Z. 47, pp. 318-324.

p. 761 Insert new reference: Storch, U. (1972). Bemerkung zu einem Satz von M. Kneser. Arch. Math. 23, pp. 403-404.

p. 761 line 5: replace "In Press" by 1995

p. 761 line 6: add 301, pp. 417–432.

p. 761 before line 5*: Insert new reference: Vogel, W.(1973). Uber eine Vermutung von D. A. Buchsbaum. *J. Alg.* 25, pp. 106–112.

Index.

p. 769 line 5, left column: 502 should be 501

p. 769 line 55, left column: the references to Vogel should be: 7, 111, 278, 301, 337, 363, 687.

p. 769 line 2, left column: add the page number 15 to the references for the word "nonzero-divisor"

p. 776 add an index item in the list after "ideal", after line 15 in the left column:

of finite colength, 234–236,272,275–279

p. 780 remove all the references to "parameter ideal"

: added 6/16/96

Ch. 10

p. 235 line 17: replace "parameter ideals for" by "ideals of finite colength on".

p. 235 line 7*: replace "is a parameter ideal for" by "has finite colength on". (this is the second occurence on line 7*!)

p. 236 line 8: replace "is a parameter ideal for" by "has finite colength on".

p. 236 line 8: replace "then it is for" by "then it has for".

p. 236 line 11*-10* : replace "is a parameter ideal for" by "has finite colength on".

p. 236 line 9*-8* : replace "is a parameter ideal for" by "has finite colength on".

Ch. 12

p. 273 line 15*-14* : replace "a parameter ideal for" by "an ideal of finite colength on".

p. 274 line 4-5 : replace "a parameter ideal for" by "an ideal of finite colength on".

p. 274 line 8* and line 6* (two occurrences) : replace "parameter ideal" by "ideal of finite colength".

p. 275 line 2 : replace "parameter ideal" by "ideal of finite colength".

p. 276 line 17 : replace "parameter ideal" by "ideal of finite colength".

p. 276 line 9* : replace "parameter ideal" by "ideal of finite colength".

p. 277 line 2*, 1* (two occurrences!) : replace "a parameter ideal for" by "an ideal of finite colength on".

p. 279 line 2 : replace "a parameter ideal" by "an ideal of finite colength".

p. 279 line 3*: replace "a parameter ideal for" by "an ideal of finite colength on".

Appendix A2:

p. 594 line 17* (display) change ϕ_1 to ϕ_2 and change ϕ_0 to ϕ_1.

p. 599 line 13 (display as on p. 594) change ϕ_1 to ϕ_2 and change ϕ_0 to ϕ_1

Appendix A3:

p. 677 line 3*. replace "has just been" by "is about to be".

p. 677 line 1*. after "1963" add ", quoted in the introduction to Hartshorne [1966b]."

: added 7/11/96

p.33 line 11. f^n should bef^d

p.43 line 4*: "algebra" should be "algebraic".

p.48 line 9*: "1992" should be"1993"

p.52 line 3: "coincident" should be "incident"

p. 58 line 3: $\mathbf{A}_k^{r+1}$ should be $\mathbf{A}^{r+1}(k)$

p. 58 line 13: "algebra" should be "algebraic"

p. 58 line 16: $\mathbf{A}_k^r$ should be $\mathbf{A}^r(k)$

p. 58 In the figure z and $x-1$ should have the same sign. The branches live in the first and third-quadrant.

p. 70 line 8 : "left" should be "right"

p.90 line 3 : more space around the vertical bar in the set definition

p.114 line 7, Ex. 3.18a : After "Show that" insert ", if k is an infinite field, then"

p.129 line 3 : $x-1$ should be $x+1$

p.136 in Exercise 4.11 the reference to A3.3 should be to A3.2.

p.156 line 2* : ";" should be "."

p.158 line 1*: At the end there should be a parenthesis ")"

p.251 line 8 : "ring" should be "reduced ring"

p.252 line 12*: R_P should be R_{P_j}

p.303 line 3: $\otimes_P$ should be $\otimes_R$

p.304 line 5: $f_i(x_1, \ldots, x_n, y_0, \ldots, y_m)$ should be $f_i(x_0, \ldots, x_m, y_0, \ldots, y_n)$

p.304 line 6: $m+1$ should be $n+1$

p.305 line 17*: "polymials" should be "polynomials"

p. 325 line 15*. line should end with a colon

p. 330 line 15 (mid of Prop). In the expression that ends with)), the second one should also be in math mode (not so tilted)

p. 333 line 7. comma before last word should be period

p. 367 line 13 "ldots" should be "..." (missing backslash in the tex

p.384 line 11 Diagram: $\exists! e$ should be $\exists! e'$

p.389 line 8 f should be ϕ

p.389 line 11 ϕ^* should be f^*

p.391 line 13* R/S should be S/R

p.405 line 15* R/P should be S/P

p.405 line 6* R/P should be S/P

p.410 line 2* after "as a" insert "deformation"

p.431 line 9* 17.4 should be 17.14

p.539 line 4*: "first" should be "middle"

p.539 line 3*: should be " $f_2 = x_1x_3 - x_2^2$

p.539 line 1*: change "$x_0 = x_1 = 0$" to "$x_1 = x_2 = 0$"

p. 754 line 8 (Herzog-Kunz reference) Should be "Herzog, J., and "

Not corrected in the Oct 96 printing

In the following "first" means first printing, "sec" means second printing (Fall 96).

Chapter 1

first 152 sec.154 Figure 5.2 There should not be any space between "in" and "$(y^2$..." otherwise it takes time (for me) to figure out that you actually mean the initial term of $(y^2-$... .

p. 251 (of first printing). First line of the pf of Cor 11.4: Prop. 11.3, which is referenced, requires the ring to be reduced.

first 256 sec.260 Corollary 11.7a and its proof: At the end of the statement of part a of the Cor, before the period, add: "modulo the group of units of R". In the fourth line of the proof of part a, replace ", so it" by:

". We have $Ru = Rv$ iff u and v differ by a unit of R, so we may identify the group of principal divisors, under multiplication, with the group $K(R)^*/R^*$. If I is any invertible divisor and Ru is a principal divisor, then $(Ru)I = uI$. Thus it"

p. 327 (of first printing). There should be an end-of-proof sign following the statement of Prop 15.4

p. 332 (of first printing). The large insert fixing the proof of Thm 15.8 was put in the wrong place: The part from "choose and expression" to "possible. Now" should be deleted.

first 528 sec 532 first paragraph after definition. Towards the end of the paragraph there is a reference to a proposition. It should be "Proposition 21.5d" not 21.4d.

p. 546 (of first printing). Exercise 21.6 should refer explicitly to the notation introduced just after Prop. 21.5.

p. 609 (of second printing) First line after the proof. "Corollary" should be "Theorem". Same in EXC A2.15

first 645 sec 652 exact sequence labeled as 0: It should be $0 \to B \to A + B \to A \to 0$ and not $0 \to A \to$ etc.

first 645 sec 652 last paragraph of Exc A3.26a $E^1(A, B)$ should be and $E^1_R(A, B)$

first 645 sec 652 last paragraph of Exc A3.26c should end with B

first 778,sec 790 Reference page for "M-sequence" should be 419ff in the first printing and 423ff in the second printing

Thanks!

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