Gauss and the 17-gon

The Greeks of the age of Euclid and Archimedes showed how to construct equilateral triangles, squares, pentagons and hexagons using just an unmarked ruler and a compass — the constructions many of us learned in school. Some larger regular polygons are also easy to make from those: by bisecting the sides of a pentagon you can build a 10-gon. But despite many attempts over the next 2000 years, no such construction could be found for many regular polygons, even the one with only 7 sides. Was this merely for lack of ingenuity?

Carl Friedrich Gauss (1777–1855) is now recognized as one of the most original and fecund scientists in history. He was only nineteen when he saw how to translate the problem of constructing regular polygons into the language of algebra, and developed an algebraic argument proving that a 17-gon *can* in fact be constructed with ruler and compass. This feat, in his very first publication, made Gauss a star and helped make up his mind about a career in mathematics — being gifted in the classics as well, he had thought of becoming a philologist.

He also showed that it is *impossible* to construct with ruler and compass a great many regular polygons, including those having 7, 9, 11 or 13 sides. In fact, Gauss soon solved completely the problem of what regular polygons are constructible: the answer is *only those whose number of sides is a product of different Fermat primes, times perhaps a power of* 2. A *Fermat prime* is a prime number that is equal to $2^{2^k}+1$ for some counting number k — a very rare property. Even today the only Fermat primes we know are those that Gauss already knew: 3, 5, 17, 257, 65537.

Gauss went on to make amazing contributions to algebra, number theory, analysis, geometry, probability and statistics and, beyond mathematics, to astronomy, cartography, surveying and geodesy, capillarity, electromagnetism, optics and more — an almost inconceivable breadth of achievement.

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