

Foliated spaces look locally like products, but their global structure is generally not a product, and tangential differential operators are correspondingly more complex. In the 1980s, Alain Connes founded what is now known as noncommutative geometry and topology. One of the first results was his generalization of the Atiyah-Singer index theorem to compute the analytic index associated with a tangential (pseudo)-differential operator and an invariant transverse measure on a foliated manifold, in terms of topological data on the manifold and the operator.

This book presents a complete proof of this beautiful result, generalized to foliated spaces (not just manifolds). It includes the necessary background from analysis, geometry, and topology. This second edition has improved exposition, an updated bibliography, an index, and additional material covering developments and applications since the first edition came out, including the confirmation of the Gap Labeling Conjecture of Jean Bellissard.



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**9**

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Global Analysis on Foliated Spaces

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# Global Analysis on Foliated Spaces

Second Edition

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