

*“Because certainty is desirable in didactic discourse — the pupil wishes to have nothing uncertain delivered to him — the teacher cannot let any problem stand, circling it from a distance, so to speak. Things must be determined at once (staked out, as the Dutch say<sup>1</sup>), and so one believes for a while that one owns the unknown territory, until another person rips out the stakes again and immediately sets them down, nearer or farther as the case may be, once again.”*

J. W. Goethe, in *Werke* (Weimar 1893), part II, vol. 11 (“Science in General”), p. 133.

## Foreword

In this second volume of *Algebra*, I have followed the same expository guidelines laid out in the preface to the first volume, with the difference that now pedagogic considerations can take a secondary role in favor of a more mature viewpoint on the content.

I imagine the reader of this second volume to be a student who already has a working knowledge of algebra and is eager to extend and deepen this knowledge in one direction or another. Thus, in sections that can to a large extent be studied independently of the rest, I have made a broader choice of presentation.

There was good reason, in my opinion, to let the material in the first volume be guided by an emphasis on fields. Thus it was natural to present in this second volume certain classes of fields having additional structure. Among these we deal first with ordered fields, in part to arouse interest in the area of real algebra, which is given short shrift in most current textbooks (though it was much esteemed in the nineteenth century and gained new momentum in the 1920s through the work of Artin and Schreier). It also seemed worth broaching certain aspects of the theory of quadratic forms.

Next, special attention is devoted to the theory of valued fields. Local fields represent today, over a hundred years after their discovery by Hensel, a completely standard prerequisite in many areas of mathematics.

Besides making an effort not to treat superficially any area once selected for coverage, I also aimed for some diversity. Thus I decided not to stay within the confines of field theory proper, but rather to include another major theory, that of semisimple algebras. In this context it seemed a matter of course to discuss the rudiments of finite group representations as well. This path to the subject, offered here instead of the more direct one opened up by Schur, is well worth the trouble,

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<sup>1</sup> Goethe writes the past participle of the Dutch verb *bepalen*, bridging the two meanings: literally ‘to plant stakes’, but in its normal usage ‘to prescribe, determine, fix, set in place.’

especially if one is interested also in questions of rationality in representation theory, as they are treated at the end of the book.

Undoubtedly this volume contains more material than can be covered in one undergraduate semester. Some sections are perhaps suitable for introductory graduate seminars. Several topics absent from average textbooks are included here, as they fit naturally with our treatment: among them we mention the Witt calculus, Tsen rank theory, and local class field theory.

My warm thanks go to all who helped in the creation of this book: the students in my course, for their invigorating interest; my faculty colleagues, for much good advice and for prodding me on with their frequent inquiries about when the book would be ready; Florian Pop, for a conversation in Heidelberg, which persuaded me to include the topic of local class field theory; Hans Daldrop, Burkhardt Dorn and Hubert Schulze Relau, for their critical reading of large portions of the manuscript, and the latter also for his careful work on the index; Bernadette Bourscheid for her efficient preparation of the original typescript; the publishers of the first German edition (1990), BI-Wissenschaftsverlag, for their renewed cooperation, and particularly the editor Hermann Engesser, for understanding and patient advice.

In the preparation of the second German edition (1997) I again benefited from suggestions, praise and criticism from colleagues, including S. Böge, B. Huppert, J. Neukirch, P. Roquette and K. Wingberg, and from the involvement of students — not only from my course but also from elsewhere — whose watchful reading led to improvements. A special thank you goes to Susanne Bosse for her professional resetting of the text in  $\LaTeX$ .

Now I am pleased to see this second volume of my work being made available in English as well. I'm very thankful to Springer New York and its mathematics editor, Mark Spencer, for his support and good advice. The translation, like that of the first volume, was done by Silvio Levy, and once again he has suggested helpful improvements to the exposition. I shall look back upon this fruitful collaboration with fondness and appreciation.

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Falko Lorenz