I have recently been working on a number of projects connected with the exterior algebra, partly motivated by the work of Green described in Chapter 7. This led me to offer a course on the subject again in the Fall of 2001, at the University of California, Berkeley. I rewrote the notes completely and added many topics and results, including material about exterior algebras and the Bernstein–Gelfand–Gelfand correspondence.

Other Books

Free resolutions appear in many places, and play an important role in books such as [Eisenbud 1995], [Bruns and Herzog 1998], and [Miller and Sturmfels 2004]. The last is also an excellent reference for the theory of monomial and toric ideals and their resolutions. There are at least two book-length treatments focusing on them specifically, [Northcott 1976] and [Evans and Griffith 1985]. The books [Cox et al. 1997] and [Schenck 2003] give gentle introductions to computational algebraic geometry, with lots of use of free resolutions, and many other topics. The notes [Eisenbud and Sidman 2004] could be used as an introduction to parts of this book.

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Notation

Throughout the text $\mathbb{K}$ denotes an arbitrary field; $S = \mathbb{K}[x_0, \ldots, x_r]$ denotes a polynomial ring; and $\mathfrak{m} = (x_0, \ldots, x_r) \subset S$ denotes its homogeneous maximal ideal. Sometimes when r is small we rename the variables and write, for example, $S = \mathbb{K}[x, y, z]$.