

**Introductory Workshop:
Noncommutative Algebraic Geometry and
Representation Theory**

January 28 to February 1, 2013

MSRI, Berkeley, CA, USA

Organizers:

Michael Artin (Massachusetts Institute of Technology - MIT)

Michel Van den Bergh* (Vrije Universiteit Brussel)

Toby Stafford (University of Manchester)

INTRODUCTORY WORKSHOP ON “NONCOMMUTATIVE ALGEBRAIC GEOMETRY AND REPRESENTATION THEORY”.

1. ORGANIZERS

- Michael Artin (Massachusetts Institute of Technology - MIT).
- Toby Stafford (University of Manchester).
- Michel Van den Bergh (Fund for Scientific Research Flanders).

2. SCIENTIFIC DESCRIPTION

Just as commutative algebra is closely related to algebraic geometry, experience has taught us that many classes of noncommutative algebras can be best understood by attaching some kind of geometric intuition to them. However, unlike the commutative case there is as yet no uniform geometric way to think about noncommutative algebras.

The workshop was part of the MSRI program “Non-commutative Algebraic Geometry and Representation Theory” which has focussed on the various especially useful geometric models for noncommutative algebras that have appeared in recent years, as well as their interactions.

The main purpose of the introductory workshop was to serve as the foundation for this parent program. Thus the workshop provided a number of short lecture series to introduce postdocs, graduate students and non-experts to some of the major themes of the program. A number of lectures were direct introductions to specific topics but other lectures were broader in scope and provided background information.

As many participants to our workshop also attended the preceding two day “Connection for Women” workshop we made sure that the programs of the two workshops were well integrated. In a number of cases the CfW lectures served as broad introductions to the more in depth lectures of our workshop.

3. PRESENTATIONS

The subjects covered in our workshop were

- Hochschild cohomology.
- Growth Functions of Infinite Dimensional Algebras
- Noncommutative algebraic geometry
- Derived Categories
- D-Module Theory
- Deformation-Quantization
- Noncommutative Resolutions of Singularities
- Noncommutative motives
- Symplectic Reflection Algebras

With hindsight we can say this was an appealing mix of topics. The workshop has attracted many participants working in related areas, many in early stages of their career

In general we have organized the lecture series in such a way that there was a natural flow of ideas going from one lecture series to the next. For example the topic “Derived categories” was scheduled early on since it is relevant for many other topics.

Our choice for the first lecture of the workshop fell on Ragnar Buchweitz since he is known to be an excellent speaker. He didn’t fail and presented a nice lecture series on Hochschild cohomology, which is roughly speaking the basis for non-commutative calculus. Buchweitz stressed in particular the relation with the center of the derived category and the still unsolved problems in this area.

Jason Bell introduced the audience to the notion of growth functions. These are important since they provide the foundation for dimension theory for non-commutative algebras. He surveyed recent progress and discussed in particular Smoktunovicz’s¹ beautiful work. He also touched on several famous unsolved problems among which was Michael Artin’s conjectural classification of division algebras of transcendence degree two, one of the motivating problems in noncommutative algebraic geometry.

Subsequently Paul Smith introduced the subject of noncommutative algebraic geometry (NCALG) itself. Although self contained his lectures could be viewed as following up on overview lectures on various aspects of NCALG during the CfW workshop by Vancliff and Zhang.

Next Amnon Yekutieli started a clear and concise introduction to derived categories, with a main emphasis on Grothendieck duality. Grothendieck duality is one of the few features of commutative algebraic geometry that can be transferred virtually unmodified to the non-commutative case.

Since deformation quantization provides important examples of noncommutative algebras which have geometric intuition attached to them it was important to have a lecture series devoted to this subject. It was Vasily Dolgushev’s task to explain this difficult material to the audience which he did admirably. After explaining the usual Maurer-Cartan formalism and stating Kontsevich celebrated formality formula (which basically started the subject) he talked about Willwacher’s spectacular recent work on graph complexes and the Grothendieck-Teichmüller Lie algebra. Despite enormous progress some key conjectures remain open though.

Before a fully packed room (obviously many non-workshop participants had sneaked in) David Ben-Zvi gave a beautiful series of lectures on D-modules. He was able to cover an enormous amount of ground (essentially covering all major theorems) without sacrificing clarity. Nice pictures and intuitive explanations made the material accessible to non-experts. This was clear from the enthusiastic comments which could be heard after the lectures.

¹Agatha Smoktunovicz was supposed to be one of the key participants in our NAGRT program but unfortunately practical matters made it impossible for her to attend.

It was noticed first by physicists and later also by mathematicians that some (commutative) singularities have a resolution which is non-commutative. Expanding on his lecture in the CfW workshop Graham Leuschke explained the concept of a non-commutative resolution in detail, in particular why its definition is the way it is. He gave many examples of noncommutative resolutions “in the real world”.

Mathilde Marcolli gave a beautiful survey of her joint work with Tabuada on noncommutative motives. Noncommutative motives were introduced by Kontsevich in a somewhat informal way. The theory was put on firm footing in the PhD thesis of Keller’s student Tabuada. Marcolli illustrated the similarities between commutative motives and noncommutative motives, in particular highlighting noncommutative analogues of Grothendieck’s standard conjectures. Intriguingly it turns out that it is not known if truly noncommutative motives exist. In other words it might be that all noncommutative motives are actually commutative, but that would be a sensational result in itself.

The last series of the conference was a survey by Maria Chlouveraki on symplectic reflection algebras. These represent concrete examples of noncommutative algebras where many of the techniques presented in the earlier lectures can be employed. They are deformations of skew group rings over polynomial algebras, they are often non-commutative resolutions, in other cases they can be studied using \mathbb{Z} -algebra as introduced in noncommutative algebraic geometry, the theory makes extensive use of D -modules etc. . . . Again Maria Chlouveraki’s lectures followed up on a broad survey given by Iain Gordon during the CfW workshop.

Organizers

First Name	Last Name	Institution
Michael	Artin	Massachusetts Institute of Technology
Michel	Van den Bergh	Vrije Universiteit Brussels
Toby	Stafford	University of Manchester

Speakers

First Name	Last Name	Institution
Jason	Bell	University of Waterloo
David	Ben-Zvi	University of Texas
Ragnar-Olaf	Buchweitz	University of Toronto
Maria	Chlouveraki	Université de Versailles
Vasily	Dolgushev	Temple University
Graham	Leuschke	Syracuse University
Matilde	Marcolli	California Institute of Technology
Sholto	Smith	University of Washington
Amnon	Yekutieli	Ben Gurion University of the Negev



Mathematical Sciences Research Institute

Introductory Workshop: Noncommutative Algebraic Geometry and Representation Theory

January 28 - February 1, 2013

Schedule

Monday, January 28, 2013			
9:00AM - 9:15AM	Simons Auditorium		Welcome
9:15AM - 10:30AM	Simons Auditorium	Ragnar-Olaf Buchweitz	Variations on Hochschild cohomology I
10:30 AM - 11:00 AM	Atrium		Tea
11:00AM - 12:15PM	Simons Auditorium	Jason Bell	Introduction to Growth and growth functions I
12:15PM - 2:00PM	Atrium		Lunch
2:00PM - 3:15PM	Simons Auditorium	Paul Smith	Introduction to non-commutative algebraic geometry I
3:15PM - 3:45PM	Atrium		Tea
3:45PM - 5:00PM	Simons Auditorium	Amnon Yekutieli	Introduction to Derived Categories I

Tuesday, January 29, 2013			
9:15AM - 10:30AM	Simons Auditorium	Ragnar-Olaf Buchweitz	Variations on Hochschild cohomology II
10:30AM - 11:00AM	Atrium		Tea
11:00AM - 12:15PM	Simons Auditorium	Amnon Yekutieli	Introduction to Derived Categories II
12:15PM - 2:00PM	Simons Auditorium		Lunch
2:00PM - 3:15PM	Simons Auditorium	Paul Smith	Introduction to non-commutative algebraic geometry II
3:15PM - 3:45PM	Atrium		Tea
3:45PM - 4:30PM	Simons Auditorium	Vasily Dolgushev	Deformation Quantization I
4:30 PM - 6:20 PM	Atrium		Reception

Wednesday, January 30, 2013			
9:15AM - 10:30AM	Simons Auditorium	David Ben-Zvi	Introduction to D-modules I
10:30AM - 11:00AM	Atrium		Tea
11:00AM - 12:15PM	Simons Auditorium	Vasily Dolgushev	Deformation Quantization II

Thursday, January 31, 2013			
9:15AM - 10:30AM	Simons Auditorium	David Ben-Zvi	Introduction to D-modules II
10:30AM - 11:00AM	Atrium		Tea
11:00AM - 12:15PM	Simons Auditorium	Graham Leuschke	Non-commutative desingularizations and MCM modules I
12:15PM - 2:00PM	Atrium		Lunch
2:00PM - 3:15PM	Simons Auditorium	Matilde Marcolli	Noncommutative motives and their applications I
3:15PM - 3:45PM	Atrium		Tea
3:45PM - 5:00PM	Simons Auditorium	Maria Chlouveraki	Symplectic reflection algebras I

Friday, February 01, 2013			
9:15AM - 10:30AM	Simons Auditorium	Jason Bell	Introduction to Growth and growth functions II
10:30AM - 11:00AM	Atrium		Tea
11:00AM - 12:15PM	Simons Auditorium	Matilde Marcolli	Noncommutative motives and their applications II
12:15PM - 2:00PM	Atrium		Lunch
2:00PM - 3:15PM	Simons Auditorium	Graham Leuschke	Non-commutative desingularizations and MCM modules II
3:15PM - 3:45PM	Atrium		Tea
3:45PM - 5:00PM	Simons Auditorium	Maria Chlouveraki	Symplectic reflection algebras II

Speakers

First Name	Last Name	Institution
Katie	Ansal di	University of Notre Dame
Mike	Artin	Massachusetts Institute of Technology
Martina	Balagovic	University of York
Sean	Ballentine	University of Maryland
Helene	Barcelo	MSRI - Mathematical Sciences Research Institute
Hanno	Becker	Universität Bonn
Charlie	Beil	Simons Center for Geometry and Physics, Stony Brook University
Jason	Bell	University of Waterloo
Georgia	Benkart	University of Wisconsin
David	Benson	University of Aberdeen
David	Ben-Zvi	University of Texas
Olga	Bershteyn	Tallinn Technical University
Bryan	Bischof	Kansas State University
Mats	Boij	Royal Institute of Technology (KTH)
Ragnar-Olaf	Buchweitz	University of Toronto
Thomas	Cassidy	Bucknell University
Kenneth	Chan	University of Washington
Harrison	Chen	UC Berkeley Math Faculty
Maria	Chlouveraki	Université de Versailles
Lars	Christensen	Texas Tech University
Lee	Cohn	University of Texas
William	Crawley-Boevey	University of Leeds
Steven	Cutkosky	University of Missouri
Hailong	Dao	University of Kansas
Galyna	Dobrovolska	University of Chicago
Vasily	Dolgushev	Temple University
Emilie	Dufresne	MSRI - Mathematical Sciences Research Institute
David	Dynerman	University of Wisconsin
Michael	Ehrig	Universität Bonn
Eleonore	Faber	University of Toronto
banafsheh	farang-hariri	Université de Nancy I (Henri Poincaré)
Jiarui	Fei	University of California
Sian	Fryer	University of Manchester
Jason	Gaddis	University of Wisconsin-Milwaukee
Iordan	Ganev	University of Texas
Emanuele	Ghedin	University of Oxford
Kenneth	Goodearl	University of California
Reiner	Hermann	Universität Bielefeld
Justin	Hilburn	University of Oregon
Alexander	Hoffnung	Temple University
Cody	Holdaway	University of Washington
Jen-Chieh	Hsiao	Purdue University
Mee Seong	Im	University of Illinois at Urbana-Champaign
Alexander	Ivanov	St. Petersburg State University
Srikanth	Iyengar	University of Nebraska
Andrew	Jaramillo	University of California
Jack	Jeffries	University of Utah
Theo	Johnson-Freyd	University of California
Tina	Kanstrup	Aarhus University

Youngsu	Kim	Purdue University
Ryan	Kinser	Northeastern University
Ellen	Kirkman	Wake Forest University
Martina	Lanini	University of Melbourne
Gail	Letzter	NSA - National Security Agency
Graham	Leuschke	Syracuse University
Matilde	Marculli	California Institute of Technology
Cristian	Martinez	University of Utah
Joanna	Meinel	Max-Planck-Institut für Mathematik
Andrew	Misseldine	Brigham Young University
Andrew	Morrison	ETH Zürich
zahra	mozafar	Isfahan University of Technology
Daniel	Murfet	University of California
Manizheh	Nafari	University of Toledo
Thomas	Nevins	University of Illinois at Urbana-Champaign
Van	Nguyen	Texas A & M University
Emily	Norton	Boston College
Adam	Nyman	Western Washington University
Masahiro	Ohno	University of Electro-Communications
Bregje	Pauwels	University of California
Aleksandr	Pavlov	University of Toronto
Jeremy	Pecharich	MSRI - Mathematical Sciences Research Institute
Alexander	Polishchuk	University of Oregon
guillaume	pouchin	University of Edinburgh
Nicholas	Proudfoot	University of Oregon
Brent	Pym	University of Toronto
You	Qi	Columbia University
Theo	Raedschelders	Vrije Universiteit Brussel
Claudiu	Raicu	Princeton University
Ali	Rajaei	Tarbiat Modares
Manuel	Reyes	Bowdoin College
Alice	Rizzardo	International School for Advanced Studies (SISSA/ISAS)
Ilan	Roth	UC Berkeley Math Faculty
Natasha	Rozhkovskaya	Kansas State University
Steven	Sam	University of California
Antonio	Sartori	Universität Bonn
Ian	Shipman	University of Michigan
Gautam	Sisodia	University of Washington
Sholto	Smith	University of Washington
Gregory	Smith	Queen's University
Elaine	So	University of Pennsylvania
Spela	Spenko	University of Ljubljana
Suresh	Srinivasamurthy	Kansas State University
Hema	Srinivasan	University of Missouri
Toby	Stafford	University of Manchester
Friederike	Steglich	Friedrich-Alexander-Universität Erlangen-Nürnberg
Greg	Stevenson	Universität Bielefeld
Catharina	Stroppel	University of Chicago
Peter	Symonds	MSRI - Mathematical Sciences Research Institute
Goncalo	Tabuada	Massachusetts Institute of Technology
Hiroyuki	Terakawa	Tsuru University

Kevin	Tucker	Princeton University
Matthew	Tucker-Simmons	University of California
Michel	Van den Bergh	Limburgs Universitair Centrum
Michaela	Vancliff	University of Texas at Arlington
monica	vazirani	University of California
Padmini	Veerapen	University of Texas
Friedrich	Wagemann	Universite de Nantes
Chelsea	Walton	Massachusetts Institute of Technology
Linhong	Wang	Southeastern Louisiana University
Ben	Webster	Northeastern University
Zhaoting	Wei	University of Pennsylvania
Jerzy	Weyman	Northeastern University
Jason	Williams	Spanish Town High
Sarah	Witherspoon	Texas A&M University
Emily	Witt	University of Minnesota Twin Cities
Amnon	Yekutieli	Ben Gurion University of the Negev
Shilin	Yu	Pennsylvania State University

Officially Registered Participant Information

Participants		117
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Gender		117
Male	67.52%	79
Female	30.77%	36
Declined to state	1.71%	2

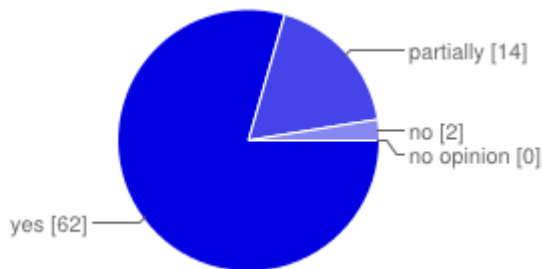
Ethnicity*		117
White	68.38%	80
Asian	18.80%	22
Hispanic	1.71%	2
Pacific Islander	0.00%	0
Black	0.85%	1
Native American	0.00%	0
Mixed	1.71%	2
Declined to state	8.55%	10

* ethnicity specifications are not exclusive

Summary [See complete responses](#)

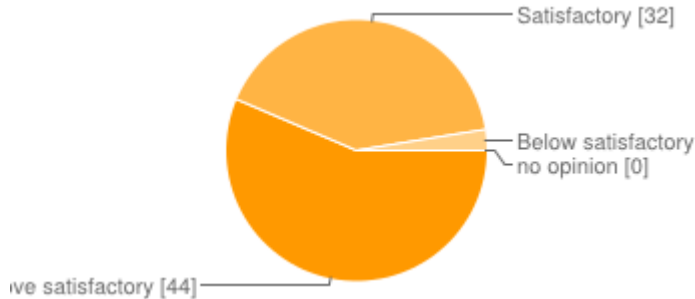
Topic presentation and organization

Did the various topics within the workshop integrate into a coherent picture?



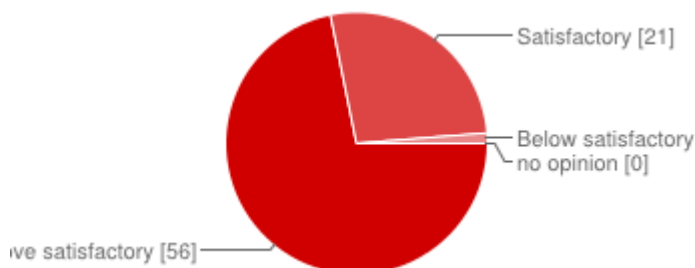
yes	62	79%
partially	14	18%
no	2	3%
no opinion	0	0%

Were the speakers generally clear and well organized in their presentation?



Above satisfactory	44	56%
Satisfactory	32	41%
Below satisfactory	2	3%
no opinion	0	0%

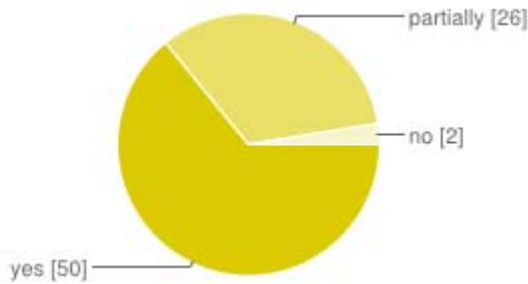
Was there adequate time between lectures for discussion?



Above satisfactory	56	72%
Satisfactory	21	27%
Below satisfactory	1	1%
no opinion	0	0%

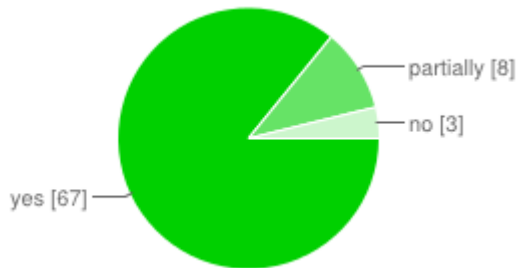
Personal assessment

Was your background adequate to access a reasonable portion of the material?



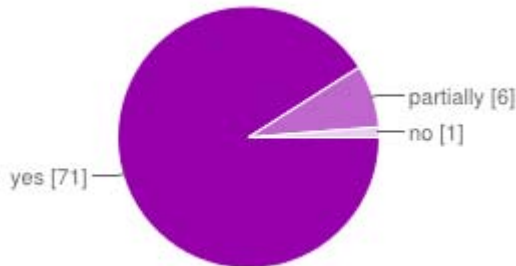
yes	50	64%
partially	26	33%
no	2	3%

Did the workshop increase your interest in the subject?



yes	67	86%
partially	8	10%
no	3	4%

Was the workshop worth your time and effort?



yes	71	91%
partially	6	8%
no	1	1%

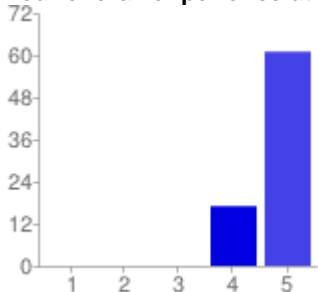
Additional comments on your personal assessment

There should be a graduate student specific workshop
 talks concerning the theme of my current investigations
 level, but ...

there were several

Many of the lectures were well pitched to a graduate student

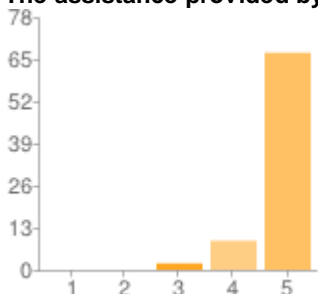
Your overall experience at MSRI



1 - Not satisfactory	0	0%
2	0	0%
3	0	0%
4	17	22%
5 - Above satisfactory	61	78%

Not satisfactory Above satisfactory

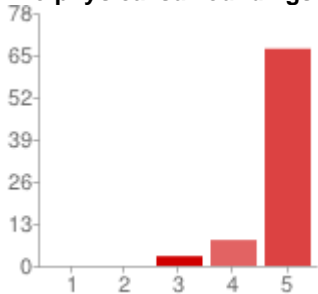
The assistance provided by MSRI staff



1 - Not satisfactory	0	0%
2	0	0%
3	2	3%
4	9	12%
5 - Above satisfactory	67	86%

Not satisfactory Above satisfactory

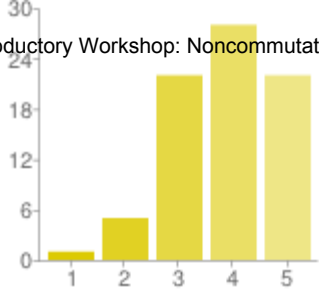
The physical surroundings



1 - Not satisfactory	0	0%
2	0	0%
3	3	4%
4	8	10%
5 - Above satisfactory	67	86%

Not satisfactory Above satisfactory

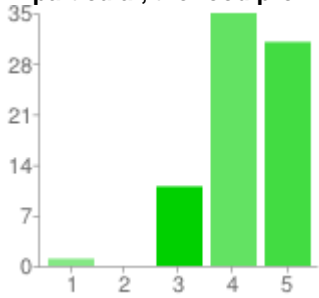
The food provided during the workshop



Not satisfactory Above satisfactory

1 - Not satisfactory	1	1%
2	5	6%
3	22	28%
4	28	36%
5 - Above satisfactory	22	28%

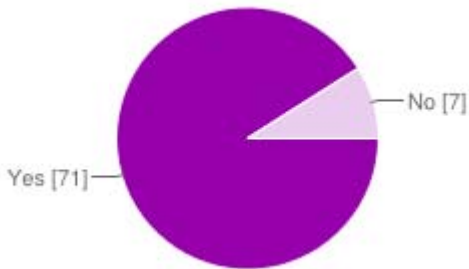
In particular, the food provided during the reception



Not satisfactory Above satisfactory

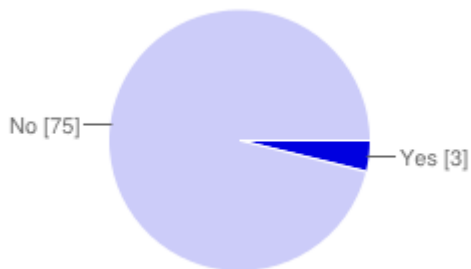
1 - Not satisfactory	1	1%
2	0	0%
3	11	14%
4	35	45%
5 - Above satisfactory	31	40%

Did you use MSRI's wireless network?



Yes	71	91%
No	7	9%

Did you experience any difficulties with the network?



Yes	3	4%
No	75	96%

If you did experience difficulties with the network, please explain:

connection.
improved

food service could be

Additional comments on the venue

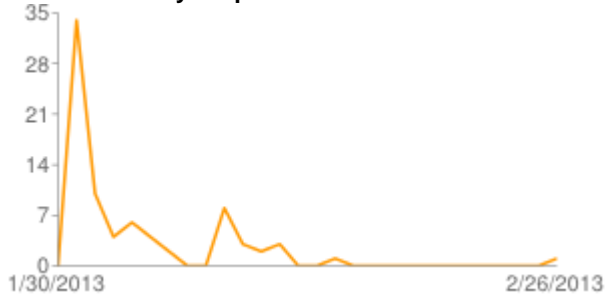
It was nice to have a charter bus. I found atmosphere in MSRI
absolutely fantastic! Again, food could have done with labelling: vegetarian, vegan, food
allergies/intolerances, etc. the temperature in t ...

Thank you for completing this survey

We welcome any additional comments or suggestions you may have to improve the overall experience for future participants.

Very
inspiring! suggestions: (1) name tags with clips or lanyards rather than pins
(2) small erasers, since the large ones are unwieldy and when they are old and bent they don't erase
well If the work ...

Number of daily responses



Introductory Workshop

Noncommutative Algebraic Geometry and Representation Theory

January 28 to February 1, 2013

Additional Survey Responses

Additional comments on the topic presentation and organization

- Idea with extra shuttle up and down the hill where MSRI stands was excellent
- there was a certain amount of unevenness in the level at which the talks were pitched. Also the schedule (60min+15min) was a bit confusing, even to some of the speakers and session chairs
- I would say that some lectures were not on an introductory level
- One person's "introductory" material can be another's "advanced" report.
- Topics were very interesting and were nicely presented with a digestible amount of material.
- Breaks between talks were too long
- nice selection
- The topic presentations were very good; they gave a quick overview of the subject. Tea breaks and meals were definitely good times to talk to various people about the lectures and about related mathematics that I have been thinking about.
- The starting time was quite early in the morning, compared to most conferences I've been to. I would have preferred to start a half hour later or so.
- The time between lectures was too much
- Great!

Additional comments on your personal assessment

- There should be a graduate student specific workshop
- there were several talks concerning the theme of my current investigations
- Many of the lectures were well pitched to a graduate student level, but some started at far too high a level for what should have been an introductory course.
- It was slightly more out of my field than expected
- i had several conversations during the workshop which were very valuable to me mathematically. i also enjoyed just meeting lots of other people and being inspired by the energy and the ideas around. thank you for this wonderful opportunity.
- nice overviews and also high quality

Additional comments on the venue

- It was nice to have a charter bus.
- I found atmosphere in MSRI absolutely fantastic!
- Again, food could have done with labelling: vegetarian, vegan, food allergies/intolerances, etc.
- the temperature in the lecture hall was either much too cold or much too warm.
- The temperature in the Simon's Auditorium left a lot to be desired. On Monday, it was far too cold, and by Friday it was far too hot. Perhaps the thermostat needs to be changed or relocated?
- Please use name tags without safety pins. The blackboards in Simons hall get very dirty towards the end of a session. Perhaps better erasers could help?
- I didn't attend the reception.
- i'd just like to compliment the lunch caterers on their clam chowder (friday). it was delicious. and, the sunsets from msri are phenomenal.
- Mick was a great assistant throughout the workshop. Thank you.

We welcome any additional comments or suggestions you may have to improve the overall experience for future participants.

- Very inspiring!
- suggestions: (1) name tags with clips or lanyards rather than pins (2) small erasers, since the large ones are unwieldy and when they are old and bent they don't erase well
- If the workshop was during summer, it was better.
- I found the food better during the second half of the week. (I liked this caterer better.)
- I'm largely satisfied with the workshop. I found lectures and speakers on the last three days more inspiring and enlightening than those during the first two days.
- Thank you very much!