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Willmore Bending Energy on the Space of Surfaces

Rob Kusner, Andrea Mondino, and Felix Schulze

The interplay between curvature and topology abounds at MSRI this spring, with Differential Geometry the theme of our semester-long program. Since it would be impossible to touch upon everything happening here now, we focus on a topic close to our recent interests: how the curvature-based Willmore bending energy can be used to turn a sphere inside out without creasing or tearing, explain the shapes of red blood cells, estimate the mass of the cosmos, and more.

Bending Energy: From Robert Hooke to Tom Willmore via Sophie Germain

Imagine a surface in space made from an ideal isotropic material that can be effortlessly stretched, but resists being bent. The "bending version" of Hooke's Law for elasticity predicts that the energy density of the surface is proportional to the sum of the squares of its principal curvatures, κ_1 and κ_2, which can be rewritten $\kappa_1^2+\kappa_2^2=(4H^2-2K)$ in terms of the mean curvature $H=\frac{1}{2}(\kappa_1+\kappa_2)$ and the Gauss curvature $K=\kappa_1\kappa_2$. The Gauss–Bonnet formula,

$$\int_\Sigma K = 2\pi\chi(\Sigma),$$

relates the total Gauss curvature to the Euler number $\chi(\Sigma)$ of a closed surface Σ in $\mathbb{R}^3$. This formula — or the corresponding relation for a surface with suitable boundary conditions — implies the *bending energy* is proportional to

$$W(\Sigma)=\int_\Sigma H^2$$

plus a topological constant. Although now named for T. J. Willmore, who in the 1960s rekindled interest in W for closed surfaces, this bending energy was introduced by Sophie Germain to explain the sand patterns seen

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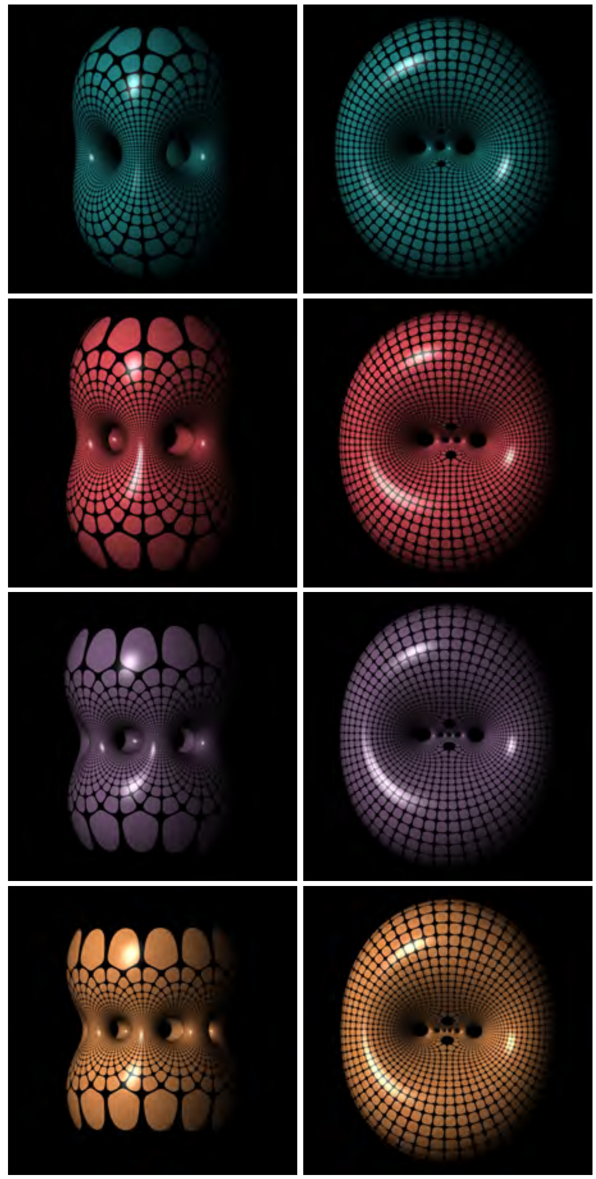

At right: Two stereographic projections of Lawson's minimal surfaces $\xi_{g,1}$ of genus $g=3, 4, 5, 6$, with principal curves drawn. These are conjectured to be the Willmore energy minimizers for each genus. (Graphics by N. Schmitt, Universität Tübingen GeometrieWerkstatt.)

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The View from MSRI

David Eisenbud, Director

I have the great pleasure of announcing some news that is particularly welcome in this era of federal budget belt-tightening: a number of long-term development efforts have come to fruition this year!

New Funding for Research

Shiing-Shen Chern Endowment. First, MSRI will receive a new $5 million endowment, named in honor of MSRI's founding Director: the Shiing-Shen Chern Endowment will be funded by Chern's son and daughter, Paul Chern and May Chu; and by one of Chern's collaborators, Jim Simons, and his wife Marilyn Simons. The endowment will help MSRI attract more of the most distinguished mathematicians to its scientific programs. These major figures in mathematics form the core of our programs, mentoring the postdoctoral fellows and other junior members, and attracting the participation of a large community of mathematicians. Starting soon, when the endowment is complete, MSRI will name at least one such "Chern Professor" each semester.

Postdoc funding. The second piece of good news is that MSRI is receiving new funding for its sought-after postdoctoral positions. Coming to MSRI is an unmatched experience for postdocs in the fields of its semester programs, and for this reason the demand for positions is very high — this year there were over 300 applications (a record) for the 2016–17 programs, despite the high degree of specialization. However, coming to MSRI for a semester has represented more and more of a financial sacrifice: the stipends for postdoctoral fellows (in sync with the NSF Postdoc stipends) have not been raised for many years, despite the substantial increase in the cost of living in the Bay Area.

Now, an anonymous donor's five-year pledge has enabled MSRI to invite almost one third more postdocs *and* to increase the stipends by 20–40%, making them competitive with excellent university positions. Causation is hard to attribute, but it is the case that 100% of the 34 postdocs to whom we made offers are planning to come to MSRI next year; I doubt that many other institutions in mathematics can make this boast.

Simons support. In addition to all this, Jim and Marilyn Simons have greatly increased their unrestricted support of MSRI; this will give MSRI new flexibility and allow us to improve both the financial circumstances of visitors and other aspects of the environment, such as computing resources and services.

Support for women with families. Targeted donations will allow us to achieve another long-desired goal: increasing the possibilities for women with children to participate in our programs, by providing adequate financial support. Several years ago, MSRI established the position of Family Services Coordinator with a generous gift from Roger Strauch to support all men and women coming to MSRI with families; now, additional funding from trustee Margaret Holen and her husband David Coulson, as well as a second gift from Vinita and Naren Gupta, will provide additional support, specifically for women, at levels corresponding to the costs of daycare or school.

David Eisenbud

Science

All this good news for the future does not eclipse our current scientific programs: the wonderful jumbo program on Differential Geometry now under way (see the cover article); the Critical Issues in Math Education workshop about observing (in video) teachers in action; the prize that we gave, jointly with the CME Group, to Douglas Diamond for his work in mathematical economics (see page 12); and, last but not least, this year's Hot Topics workshop, on Wall Crossing and Cluster Algebras, an extraordinary application of ideas from string theory to an expanding field that used to seem very distant from that subject.

Public Understanding of Math

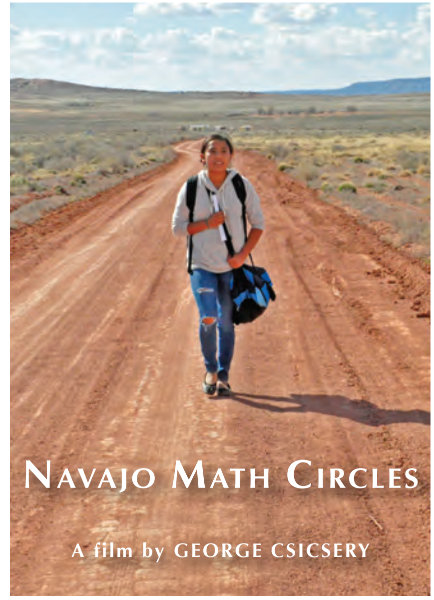
Our outreach programs are buzzing as well. Building on the great public enthusiasm for the National Math Festival that we ran jointly with the Institute for Advanced Study in Washington, DC, last April, we're planning a second festival for April 2017. The Simons Foundation and the Kavli Foundation have already pledged their support, and we expect that a number of other foundations and companies will soon join them.

The Mathical Book Prize, our award for outstanding math-related books for kids of all ages, is now a partnership with the National Council of Teachers of Mathematics and the National Council of Teachers of English, vast organizations whose collaboration will help us make the prize known and get the winning books into the hands of children throughout the country, particularly in low-income communities. The National Association of Math Circles is pursuing similar goals. Numberphile, the very popular YouTube video channel created by journalist Brady Haran with MSRI support, now has over 1.6 million subscribers, and a brand new subseries on "Tadashi's Toys" (Tadashi Tokieda is an applied mathematician and a truly great popularizer). More details about all of these programs are in the update on page 4.

Community Does It

In all these activities, our purpose is to support the mathematical sciences community in both its research mission and its relation to the public, but in this we depend on community participation: we host, rather than perform. And of course we depend on community support in many other ways, including financial — the donors of large gifts are very concerned with knowing that they are really supporting the things that are important to the mathematics community, and this is partly measured by the number of contributors in our community. I want to thank our many supporters, listed in the Annual Report on page 14. Your gifts impact our work at MSRI and help us with raising additional funds from private sources to keep growing our programs. Thank you!

Math Circles: On Screen, In Print



Math Circles have been having their moment in the sun lately, both in print and on the screen. MSRI's support program for Math Circles, the National Association of Math Circles (NAMC), was delighted to have some of its members interviewed for a feature article in the March issue of *The Atlantic Monthly*, "The Math Revolution." The article explores why the number of American teens who excel at advanced math has surged, and Math Circles naturally play a starring role! The public interest in Math Circles triggered by the *Atlantic* article led to an *Education Week* interview with NAMC Director Diana White and Associate Director Brandy Wiegers.

The new documentary film *Navajo Math Circles* is now traveling around the country for screenings and is available for DVD purchase. Several of the students and teachers from the film attended the world premiere at the Joint Mathematics Meetings in January, and will also be appearing at the Navajo Nation Museum in Arizona in April.

To read the *Atlantic* and *Education Week* articles, visit tinyurl.com/MathRevolution and tinyurl.com/EduWeek-MathCircles. To learn more about Navajo Math Circles and screenings near you, you can see www.navajomathcirclesfilm.com.

Focus on the Scientist: Sun-Yung Alice Chang

Sun-Yung Alice Chang, the Eugene Higgins Professor of Mathematics at Princeton University, has made outstanding contributions both in geometry and analysis.

Sun-Yung Alice Chang

Alice was born in China in the ancient capital city of Xi'an. During the period of the Chinese Revolution, her family moved to Hong Kong and then to Taiwan when she was two. She grew up in Taiwan and attended National Taiwan University. Later she moved to the U.S. where she obtained her Ph.D. in mathematics at UC Berkeley. There she met her husband, Paul Yang, who was a graduate student working on geometric problems.

Alice held positions at the State University of New York at Buffalo, the University of California, Los Angeles, and the University of Maryland before moving to Princeton University in 1998. Her deep mathematical contributions have been widely recognized, including a plenary talk at the 2002 International Congress of Mathematicians in Beijing as well as an American Mathematical Society Colloquium talk in 2004. In 1995, she received the Ruth Lyttle Satter Prize in Mathematics. The prize is awarded every two years to a woman who has made an outstanding contribution to mathematics research in the previous five years. Alice is currently a member of the US National Academy of Sciences.

Alice's first interests came from harmonic analysis, as shown in her Ph.D. work, where she studied the boundary behavior of bounded analytic functions on domains. Her deep analytical intuition has shaped her view in partial differential equations, establishing a solid bridge between analysis and geometry.

Her interest in geometry initially came from mathematical collaboration with her husband. They worked on the problem of prescribing Gaussian curvature on the sphere, where a crucial ingredient is the study of extremals for a Sobolev-type inequality. Alice also got involved in conformal geometry, which is the natural higher dimensional analogue for the geometry of surfaces, and spectral invariants in particular.

One of Alice's main achievements is the understanding of conformal invariants in higher dimensions. While Gauss curvature is the canonical invariant for surfaces, Alice has made a deep contribution to the understanding of fourth-order Q-curvature, which is the natural replacement in four dimensions since it controls both the geometry and the topology of such a manifold. She has also considered fully nonlinear versions of scalar curvature in conformal geometry, which in particular yields classification results for four-dimensional manifolds.

In her own words: "I have always felt that mathematics is a language like music. To learn it systematically, it is necessary to master small pieces and gradually add another piece and then another. Sitting in a good mathematics lecture is like sitting in a good opera. Everything comes together. They get to the heart of the problem and I enjoy it!"

Alice has succeeded in transmitting this passion for mathematics and the appreciation of its beauty to her students (more than fifteen already). Being Alice's student and collaborator has engendered my deep admiration and respect for her, both as a mathematician and as a woman.

— María del Mar González Nogueras

Public Understanding of Mathematics

Mathical Book Prize

In January, MSRI and the Children's Book Council announced the 2016 winners of the Mathical: Books for Kids from Tots to Teens youth literature prize. The Mathical Book Prize cultivates a love of mathematics in the everyday world. Each year's winners and honor books join a selective and ever-growing list of new and previously published fiction and nonfiction titles for kids of all ages.

This year's award winners are *8: An Animal Alphabet*, by Elisha Cooper; *Max's Math*, by Kate Banks with art by Boris Kulikov; and graphic novel *Secret Coders*, by Gene Luen Yang and Mike Holmes. Gene Luen Yang, co-creator of *Secret Coders*, has also been named the fifth National Ambassador for Young People's Literature by the Children's Book Council, Every Child a Reader, and the Library of Congress. For a full list of current and past winners, visit mathicalbooks.org.

National Math Festival

At the USA Science and Engineering Festival in April, MSRI announced that the National Math Festival will return to the nation's capital on Saturday, April 22, 2017! The festival will take place at the Walter E. Washington Convention Center in downtown Washington, DC. Stay tuned for more announcements over the coming months about featured speakers and activities as the festival kicks into gear! For more news about the 2017 Festival (as well as about math events around the U.S. and resources for math lovers of all ages), visit our brand new website: www.nationalmathfestival.org.

Illustrative Mathematics

Math educators around the U.S. can participate in a virtual lecture series that runs through May 2016. Developed in partnership with Illustrative Mathematics, the "Speaking of Mathematics Education: Productive Conversations with Families" webinars feature exclusive content from an education forum held in conjunction with the 2015 National Math Festival. Presenters include William McCallum (University of Arizona), Suzanne Wilson (University of Connecticut), Deborah Ball (University of Michigan), and Peggy Brookins (National Board for Professional Teaching Standards). Archived webinars are also available for viewing. For more info, visit tinyurl.com/MathEducationWebinar.

Tadashi's Toys

Brady Haran and Tadashi Tokieda

Filmmaker George Csicsery and journalist Brady Haran, creator of the MSRI-supported YouTube channel Numberphile, have been making a series of videos featuring Stanford professor Tadashi Tokieda's mathematical toys, which have been entertaining viewers of all ages. If you haven't seen "Tadashi's Toys" yet, you may be pleasantly surprised: objects you can make in a matter of minutes at home, which illustrate principles of mathematics, will delight both children and seasoned mathematicians. To see videos demonstrating Tadashi's inventions, visit msri.org/toys.

Not on the Test

MSRI continues to partner with Berkeley City College for the "Not on the Test: The Pleasures and Uses of Mathematics" lecture series. September 2015 featured Barbara Simons, Board Chair of Verified Voting, speaking on "Internet Voting: Wishful Thinking?" October's talk on "The Math and Science of Pixar Films" by Danielle Feinberg, Director of Photography at Pixar Animation Studios, was extremely well attended, with over 300 attendees! February and March featured talks by two National Math Festival alumni: Kimberly Bryant of the nonprofit Black Girls Code shared her work to promote computer science education for girls of color; and Mariel Vazquez of the University of California, Davis, taught the audience about how using mathematics and computational tools is critical for her research in microbiology, as most cells in our bodies contain over two meters of DNA, which tends to knot in confined spaces. You can find videos of many of the talks at msri.org/e/not-on-the-test.

Harmonic Series Music Concerts

MSRI has welcomed the public to enjoy our free Harmonic Series music concerts throughout the year, and February's performance by the Left Coast Chamber Ensemble featured the music of Debussy, Schumann, and an original work by pianist Eric Zivian. Additional concerts in 2015–16 featured The Honey-Doos and the Galax Quartet. News about upcoming concerts in 2016–17 will be released in early fall.

Differential Geometry — Willmore Bending Energy

(continued from page 1)

on Chladni's vibrating plates. Napoleon had set a prize in 1808 for the best solution, and the only relevant submission — Germain's — was judged incomplete. Poisson then submitted his own solution and was awarded the prize; curiously, he also noted that the K part of the bending energy density is a "divergence term" that does not affect the Euler–Lagrange equation for equilibrium — or the corresponding wave equation for vibration — foreshadowing later versions of the Gauss–Bonnet relation by many decades!

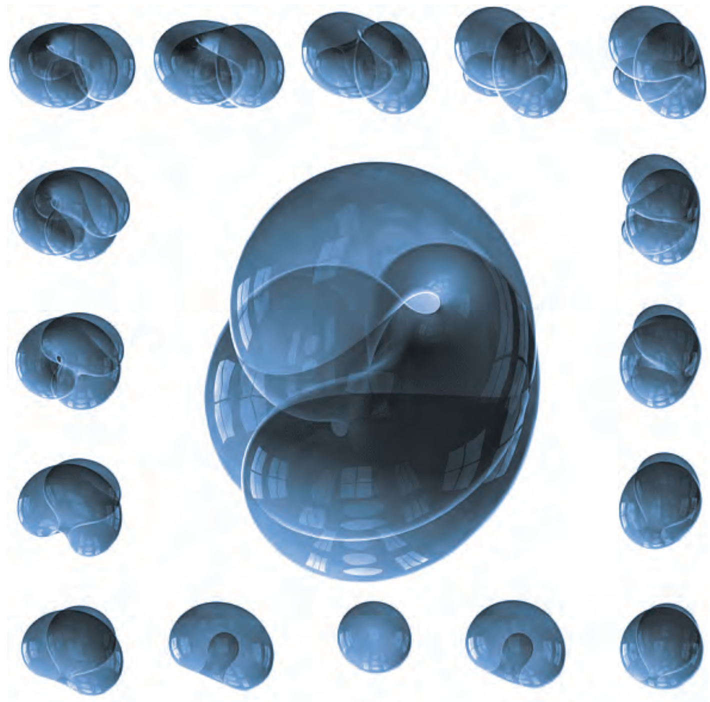
By the 1920s, W. Blaschke and his students investigated what they called *conformal area* whose density $\frac{1}{4}(\kappa_1-\kappa_2)^2 = H^2 - K$ is not only scale-invariant, but also unchanged by Möbius transformations of space. It follows that $W(\Sigma) = \int_\Sigma H^2$ is Möbius-invariant for closed surfaces, since $\int_\Sigma K$ is a topological invariant. Integrating the inequality $H^2 \geqslant K$ over an immersed sphere and using Gauss–Bonnet immediately gives $W \geqslant 4\pi$. In fact — as Willmore himself noted — for any closed Σ immersed in $\mathbb{R}^3$, one can instead use the better inequality $H^2 \geqslant K_+ = \max(K, 0)$ along with H. Hopf's observation that "one can lay Σ on a table in any direction" to conclude $W(\Sigma) \geqslant \int_\Sigma K_+ \geqslant 4\pi$, with equality only for round spheres.

W-Critical Surfaces and Minimal Surfaces in 3-Sphere and 3-Space

The main sources of W-minimizing or W-critical surfaces are minimal ($H \equiv 0$) surfaces in $\mathbb{R}^3$ and $\mathbb{S}^3$. In the 1970s, J. Weiner investigated the stability of W-critical surfaces, rediscovering something known to Blaschke: stereographic images $\Sigma \subset \mathbb{R}^3$ of minimal surfaces $\tilde{\Sigma} \subset \mathbb{S}^3$ — in particular, the plethora constructed by H. B. Lawson that decade — are always W-critical, with $W(\Sigma) = \text{area}(\tilde{\Sigma})$.

Robert Bryant (MSRI Director 2007–13, current AMS President) then proved a classification theorem for immersed W-critical spheres in $\mathbb{R}^3$: using Hopf's holomorphic differential techniques, as well as ideas going back to Darboux, Cartan, and Chern (founding Director of MSRI), he showed these must arise from "conformally compactifying" complete minimal surfaces with planar embedded ends; in particular, a round sphere (the conformal compactification of a flat plane) is the only *embedded* W-critical sphere in $\mathbb{R}^3$, and further, the critical values of W on the space of immersed spheres are quantized in multiples of 4π. Many examples of minimal spheres with planar embedded ends were soon constructed, not only by Bryant, but also by C. K. Peng and the first author, as well as higher genus examples by C. Costa and N. Schmitt.

Bryant's result is related to an important inequality for W proven by P. Li and S.-T. Yau, and sharpened by the first author: if d "sheets" of a closed immersed surface Σ pass through a point of $\mathbb{R}^3$, then $W(\Sigma) \geqslant 4\pi d$; equality occurs only if there is a complete minimal surface Σ' in $\mathbb{R}^3$ with d embedded planar ends — or "sheets at ∞" — with a Möbius transformation taking Σ' to Σ (and ∞, to the d-uple point on Σ).



One example of a minimax sphere eversion, shown around the border of the image. (Courtesy of J. M. Sullivan.)

Sphere Eversion and the Smale Conjecture (Now the Hatcher Theorem) via W-Flow

It is a remarkable fact that a sphere $\mathbb{S}^2$ in $\mathbb{R}^3$ can be *everted* — that is, turned inside out, without tearing or creasing. Existence of a sphere eversion is a consequence of Smale's theorem (proven shortly before he joined the UC Berkeley faculty in 1960) that any two immersed spheres in $\mathbb{R}^3$ are *regularly homotopic* — that is, the space of immersed $\mathbb{S}^2$ in $\mathbb{R}^3$ is path connected. For several years it remained a mystery how to explicitly carry this out, until A. Shapiro, B. Morin, and Bill Thurston (MSRI Director, 1992–97) each invented their own sphere eversions.

The *minimax sphere eversion* using Willmore flow on the space of immersed spheres was conceived by the first author while a grad student at UC Berkeley in the early 1980s. (This is described in detail at http://www.gang.umass.edu/~kusner/other/minimax.pdf and is illustrated in the border of the image above.) Its animation was initiated by the first author with K. Brakke, G. Francis, and J. M. Sullivan during a 1992 MSRI workshop, and debuted at the 1998 ICM in Berlin as a short film *The Optiverse* (which you can find on YouTube). Sullivan also discusses sphere eversions further at http://www.math.uiuc.edu/~jms/Papers/isama/bw/.

The seminal motivation for the minimax sphere eversion was an analytic proof — using a modified Willmore gradient flow — of the Smale conjecture:

> The space of *embedded* spheres in $\mathbb{R}^3$ deformation retracts to the subspace of round spheres, and thus is contractible.

An equivalent formulation in terms of unknotted loops in $\mathbb{R}^3$ had just been proved by A. Hatcher using combinatorial surgery methods, and in light of Bryant's then-recent result that the only embedded W-critical sphere in $\mathbb{R}^3$ is round, this approach seemed promising. Unfortunately, the lack of a maximum principle is a huge obstacle to using this fourth-order W-flow directly: besides work of E. Kuwert and R. Schätzle with the strong assumption that $W < 8\pi$, little is known about how to preserve embeddedness under the flow; there are even examples of initial immersed spheres which become singular in finite time, so — despite compelling computer animations and numerical simulations — more remains to be done to put the minimax sphere eversion via W-flow on a firm analytic foundation.

The Willmore Conjecture and the Willmore Problem

One impetus for the MSRI research program in Differential Geometry this spring was the recent proof by F. C. Marques and A. Neves of the Willmore conjecture:

> The W-minimizing torus $\Sigma_1 \subset \mathbb{R}^3$ is a stereographic image of the Clifford minimal torus in $\mathbb{S}^3$ with area $2\pi^2 = W(\Sigma_1)$.

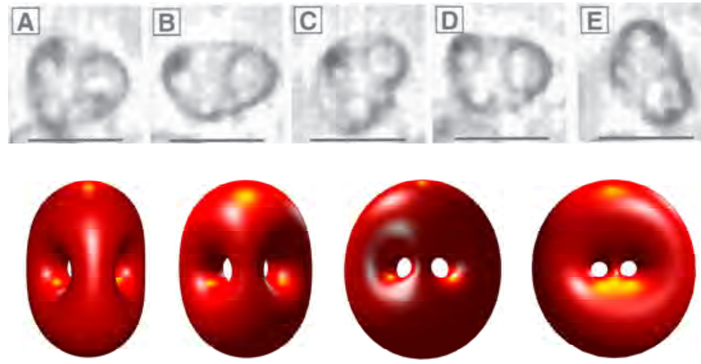
In his 1980s lecture "Three Obdurate Conjectures in Differential Geometry" at UC Berkeley, Bob Osserman (MSRI Deputy Director 1990–95, Special Projects Director 1995–2011) alluded to the unexpected difficulty and richness of this conjecture by calling it "the sleeper" among the three. As noted above, the round sphere achieves the absolute minimum $W = 4\pi$ for closed surfaces, and L. Simon later showed that for higher genus surfaces the absolute minimum is strictly greater. The punchline of the Marques–Neves argument is F. Urbano's index characterization of the minimal Clifford torus, but their main contribution is a refinement of the Almgren–Pitts min-max methods in geometric measure theory, allowing them to show (for instance) that any minimal surface in $\mathbb{S}^3$ — other than a great 2-sphere or Clifford torus — must have area strictly greater than $2\pi^2$.

The general Willmore problem is to find the W-minimizer of fixed topology — intrinsic diffeomorphism type or regular homotopy class — among closed immersed surfaces in $\mathbb{R}^3$ or $\mathbb{R}^n$. The invariance of W under the noncompact group of Möbius transformations makes this problem delicate. The existence of higher genus embedded W-minimizing surfaces in $\mathbb{R}^3$ was established in work over the past three decades by Simon, and by M. Bauer and Kuwert, using ideas of the first author, who has also conjectured:

> The Lawson surface $\xi_{g,1}$ is the lowest-area minimal surface of genus g in $\mathbb{S}^3$, and it stereographically projects to the W-minimizer of genus g in $\mathbb{R}^3$.

Some low genus Lawson surfaces are shown earlier in the figure on the cover.

For regular homotopy classes that do not contain embedded surfaces, finding an immersed W-minimizing surface representative remains an open problem, with one exception: Bryant and the first author constructed explicit immersed real projective planes, the simplest of which realizes a W-minimizing Boy's surface B in $\mathbb{R}^3$ with $W(B) = 12\pi$ (this surface is well known to anyone who has visited Oberwolfach). For immersed Klein bottles in $\mathbb{R}^3$ there are three regular homotopy classes, but only the class of B connect sum with itself (or the mirror image) is known to contain a



Images **A–E** show observations of conformal deformation of a genus-2 phospholipid vesicle (scale bar = 10 μm). The bottom series shows computed images of the deformation. (**A–E** from X. Michalet and D. Bensimon, Observation of stable shapes and conformal diffusion in genus 2 vesicles, Science **269** (1995), 666–668. Reprinted with permission from AAAS. Bottom series from Jingmin Chen's thesis.)

W-critical surface, the stereographic image of another Lawson minimal surface from $\mathbb{S}^3$, also conjectured to be the W-minimizer.

The Marques–Neves solution to the Willmore conjecture uses a clever degree argument depending on the orientability of the surface, and their tools from geometric measure theory rely on embeddedness and codimension-1 in an essential way. It is unclear if their methods can be adapted to study surfaces in higher codimension. For example, the corresponding conjecture remains open for tori in $\mathbb{R}^n$, as does the following even more basic conjecture of the first author:

> Any closed surface Σ in n-space *not* diffeomorphic to $\mathbb{S}^2$ has $W(\Sigma) \geqslant 6\pi$, with equality if and only if Σ comes from stereographic projection $V \subset \mathbb{R}^4$ of the Veronese minimal projective-plane in $\mathbb{S}^4$.

Note that V and its mirror image solve the Willmore problem for the two regular homotopy classes of embedded projective planes in $\mathbb{R}^4$, by work of Li and Yau.

Extending the Bending Energy to a Frame Energy

The Willmore bending energy W has been discussed above to study the space of immersed surfaces in $\mathbb{R}^n$. A natural extension of W is a new energy introduced by the second author and T. Rivière for *framed immersions* of a surface into space: the *frame energy* is essentially the Dirichlet energy for tangent frames on a surface Σ immersed in $\mathbb{R}^n$. The frame energy is bounded below by $W(\Sigma)$, and it is much more coercive than W: indeed, an upper bound on the frame energy prevents the degeneration caused by the action of Möbius transformations of space or the degeneration of the conformal class of the underlying surface Σ.

T. Rivière and the second author have proven a Willmore-type inequality for any codimension: the frame energy of any framed torus Σ immersed in $\mathbb{R}^n$ is at least $2\pi^2$, with equality if and only if $n \geqslant 4$ and there is a homothety carrying Σ to the square Clifford torus $S^1 \times S^1 \subset \mathbb{R}^4 \subset \mathbb{R}^n$ endowed with the standard orthonormal frame $(\partial_\theta, \partial_\varphi)$, up to a fixed rotation. The frame energy may also be used to find canonical representatives for regular homotopy classes: for instance, in each of the two regular homotopy classes of immersed tori in $\mathbb{R}^3$, there exists a smooth minimizer of the frame energy. Recently, Rivière extended the frame energy to immersions of spheres into $\mathbb{R}^m$ and used it as a viscosity term to investigate the first author's min-max proposal for constructing W-critical spheres in $\mathbb{R}^3$.

The Canham–Helfrich Model and Conformal Diffusion for Vesicles

A *vesicle* is a lipid bilayer enclosing a fluid region, typical of cell membranes or the endoplasmic reticulum within a cell. In vesicle models proposed by P. Canham and W. Helfrich in the 1970s, between these two lipid layers lies an embedded surface $\Sigma \subset \mathbb{R}^3$ critical for the modified bending energy,

$$W_{H_0}(\Sigma) = \int_\Sigma (H - H_0)^2,$$

subject to constraints on the surface area $A(\Sigma)$ and enclosed volume $V(\Sigma)$. Including the "spontaneous curvature" term H_0 amounts to varying the original bending energy $W(\Sigma)$ while imposing a third constraint on the total mean curvature $M(\Sigma) = \int_\Sigma H$, which geometrically accounts for the difference in areas of the two vesicle layers.

The Canham–Helfrich problem is to minimize $W(\Sigma)$ among embedded surfaces $\Sigma \subset \mathbb{R}^3$ of a given topological type subject to these three constraints. The Möbius invariance of W means it suffices to impose the two scale-invariant constraints

$$\nu(\Sigma) = 36\pi \frac{V(\Sigma)^2}{A(\Sigma)^3} = \nu \qquad m(\Sigma) = \frac{M(\Sigma)^2}{4\pi A(\Sigma)} = m$$

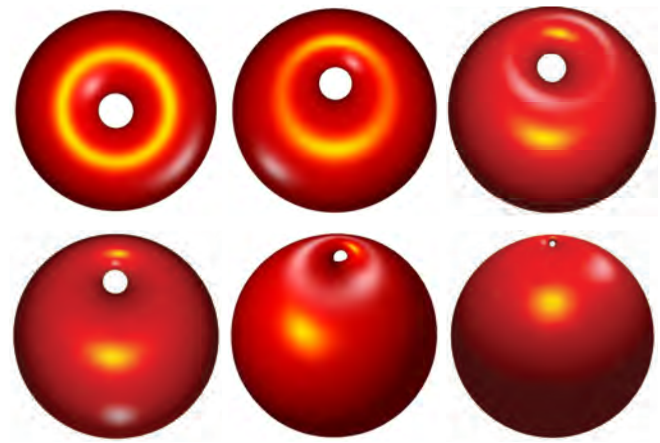
on the isoperimetric ratio and normalized mean curvature when minimizing $W(\Sigma)$. Recall that the *isoperimetric inequality* asserts $\nu(\Sigma) \leqslant 1$, with equality if and only if Σ is a round sphere.

For a generic surface $\Sigma \subset \mathbb{R}^3$, a 3-parameter family of Möbius transformations (inversions in unit spheres centered at points not on Σ) changes its shape. Thus, by the implicit function theorem, for regular values of the two constraints ν and m, one expects a 1-parameter family of solutions to the Canham–Helfrich problem. The possibility of such shape-changing families was pointed out in the first author's early work with L. Hsu and J. M. Sullivan computing W-minimizing surfaces; dubbed "conformal diffusion" by physicist U. Seifert, this phenomenon was experimentally confirmed for real — but rather rare — lipid vesicles of genus 2 in the École Normale Supérieure biophysics laboratory of D. Bensimon and X. Michalet soon afterward. (See the figure at the top of the previous page.)

The Canham Problem: Minimizing Energy with an Isoperimetric Constraint

The simpler Canham model for vesicles ignores the constraint on $M(\Sigma)$. Thus it is both natural and mathematically more tractable to consider the analogous Canham problem: minimizing $W(\Sigma)$ under the isoperimetric constraint $\nu(\Sigma) = \nu$ alone.

There is a "tautological" solution Σ to the Canham problem for genus $g > 0$: Let Σ_g be any (unconstrained) W-minimizing surface of genus g in $\mathbb{R}^3$, and let ν_g denote the smallest value of the isoperimetric ratio $\nu(\mu(\Sigma_g))$ for its image under any Möbius transformation μ. Note (by the isoperimetric inequality) that $\nu_g < 1$, and also $\nu(\mu(\Sigma_g))$ tends to 1 as μ becomes more extreme, with the surface $\mu(\Sigma_g)$ limiting to a round sphere (as a "microscopic handle" of genus g "vanishes"). Thus for any ν in the interval $[\nu_g, 1)$, we simply take $\Sigma = \mu(\Sigma_g)$ for some Möbius transformation μ with $\nu(\mu(\Sigma_g)) = \nu$: since W is Möbius invariant, this Σ achieves the minimum value $W(\Sigma) = W(\Sigma_g)$.

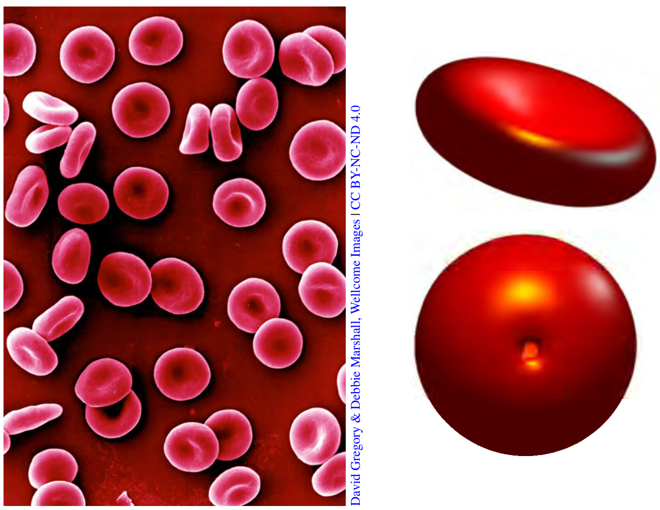


Dupin cyclides solve the genus-1 Canham problem. (From J. Chen's thesis.)

For example, when $g = 1$, the solution to the Willmore conjecture lets one explicitly compute $\nu_1 \approx 0.13$ to be the isoperimetric ratio realized by the stereographic projection of the square Clifford torus as a surface of revolution in $\mathbb{R}^3$; this gives a large interval $[\nu_1, 1)$ for which the Canham problem is solved by Dupin cyclides. (See the figure above; the first author's recent work with J. Chen and T. Yu suggests that these solutions are also unique for this interval of isoperimetric ratios.)

For $g = 0$, we have seen the W-minimizer Σ_0 is a round sphere; since Möbius transformations preserve round spheres, $\nu_0 = 1$ is the only isoperimetric ratio where the preceding "tautological" approach works. J. Schygulla overcame this difficulty by applying Möbius transformations to a perturbation of the inverted catenoid instead: for each ν in $(0, 1]$, this gives a surface Σ with $\nu(\Sigma) = \nu$ and $W(\Sigma) < 8\pi$; he then used Simon's "ambient approach" to show there exists a genus-0 solution of the Canham problem for the entire interval $(0, 1]$. The solution is not explicit, nor is uniqueness known (except for the round sphere at $\nu = 1$), but it is believed to be rotationally invariant, shaped like a red blood cell for intermediate values of ν, and like a "gastrula" for ν near 0.

In collaboration with L. Keller and Rivière, the second author used Rivière's "parametric approach" to extend the range of isoperimetric ratios for which the higher genus Canham problem is solvable to an open interval containing $[\nu_g, 1)$. While the shape of an unconstrained W-minimizer Σ_g for $g > 1$ is only conjectural, some recent numerical work of Chen and Yu with the first author indicates the optimal Möbius transformation one applies to Σ_g to estimate ν_g. They also have explored the shapes of Canham-minimizers with small values of the isoperimetric ratio: these have $W \approx 8\pi$ and look like doubled spheres joined by small necks; however, the distribution of necks is unclear. A simple yet important open question that should lead to a resolution of the higher genus Canham problem is to decide whether the strict inequality $W(\Sigma) < 8\pi$ can be achieved by surfaces of genus $g > 0$ for any $\nu(\Sigma)$ near 0.



Observed and computed red blood cells. At left, scanning electron micrograph of normal blood cells; at right, computed images showing normal (top image) and gastrula-like vesicle (bottom image). (Computed images from J. Chen's thesis.)

The Hawking Mass and the Riemannian Penrose Inequality

In general relativity it is natural to model the asymptotic spatial geometry of an isolated gravitational system by the end N of a Riemannian 3-manifold whose metric is asymptotically Schwarzschild. The effective mass of the "black hole" represented by this asymptotic metric can be computed by a flux integral at infinity which defines m_{ADM}, the *ADM mass* for such an end. The *Hawking mass*,

$$m_H(\Sigma) = (16\pi)^{-3/2} A(\Sigma)^{1/2} \big(16\pi - 4W(\Sigma)\big),$$

which is a proxy for the mass enclosed by a surface Σ within a spatial slice of spacetime, also makes use of the bending energy $W(\Sigma) = \int_\Sigma H^2$. While m_H is well-defined for all closed immersed surfaces, the mass it measures is "too small" on most of them: indeed, as observed earlier, if Σ is a closed surface in Euclidean space, then $W(\Sigma) \geqslant 4\pi$ — and so $m_H(\Sigma) \leqslant 0$ — with equality only for round spheres.

It is interesting to compare these two notions of mass. Using R. Geroch's observation that $m_H(\Sigma)$ should evolve monotonically under inverse mean curvature flow, G. Huisken and T. Ilmanen have shown that $m_H(\Sigma) \leqslant m_{ADM}$ for any outward minimizing Σ in N, yielding a proof of the long-conjectured Riemannian Penrose inequality.

A related concept is the *center of mass* for an isolated gravitational system modeled by an asymptotically Schwarzschild end N. By proving there exists a unique foliation of the end by constant mean curvature surfaces, asymptotic to a foliation by concentric spheres, G. Huisken and S.-T. Yau defined such a center. Their foliation is more closely related to yet another notion, the *isoperimetric mass*, and this raises the question of whether there exists an analogous center related to the Hawking mass. Together with T. Lamm and J. Metzger, the third author has proposed maximizing $m_H(\Sigma)$ — or equivalently minimizing $W(\Sigma)$ — among surfaces Σ with given area $A(\Sigma)$. They show there exists a family of constrained W-critical surfaces — in fact, local minimizers — that foliates the asymptotic region of N such that the leaves are asymptotic to centered spheres; it remains an open question if such foliations define a unique asymptotic center, and whether their leaves are global minimizers for W.

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website www.msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are **March 15**, **October 15**, and **December 15**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see http://tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by **March 15**, **October 15**, or **December 15** for review at the upcoming SAC meeting. See http://tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several 2-week long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by **March 15**, **October 15** or **December 15** for review at the upcoming SAC meeting. See http://tinyurl.com/msri-sgs.

Named Postdocs in Differential Geometry — Spring 2016

Strauch Postdoc

Heather Macbeth

Heather Macbeth is this spring's Strauch Postdoctoral Fellow. Heather received her undergraduate degree from the University of Auckland, New Zealand, in 2009, and a Part III from the University of Cambridge in 2010. She was a graduate student at Princeton University under the supervision of Gang Tian, receiving her Ph.D. in June 2015. Since July 2015 she has been a Moore Instructor at MIT. Heather's research is in Kähler geometry and geometric analysis. She has studied the existence problem for Kähler–Einstein metrics on Fano manifolds, extending the existence criteria in terms of alpha-invariants due to Tian. Her other projects include work on the Yamabe invariant, conformal geometry, and projective geometry.

The Strauch Fellowship is funded by a generous annual gift from Roger Strauch, Chairman of The Roda Group. He is a member of the Engineering Dean's College Advisory Boards of UC Berkeley and Cornell University, and is also currently the chair of MSRI's Board of Trustees, on which he has served for more than 15 years.

Viterbi Postdoc

Chengjian Yao

Chengjian Yao is the Viterbi Postdoctoral Fellow in Differential Geometry. Chengjian received his undergraduate degree from the University of Science and Technology of China in Hefei. In May 2015, he obtained his Ph.D. from Stony Brook University under the supervision of Xiuxiong Chen. He is currently on leave from the Chercheur postdoctoral position at Université Libre de Bruxelles in Belgium. Chengjian's research has focused on the study of the existence and regularities of conical Kähler–Einstein metrics on compact Kähler manifolds. He was able to smoothly approximate conical Kähler–Einstein metrics with singularities along a big class of simple normal crossing divisors, using the classical Aubin–Yau continuity path. Together with his collaborators Spotti and Sun, they affirmed the Yau–Tian–Donaldson conjecture in the case $\mathbb{Q}$-Gorenstein smoothable Fano-varieties. More recently, he has worked on a geometric flow, called definite triple flow, to try to attack one of Donaldson's questions which aims to give a symplectic characterization of a hyper-Kähler 4-manifold.

The Viterbi Endowed Postdoctoral Scholarship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.

Huneke Postdoc

Andrea Mondino

Andrea Mondino is the Huneke Endowed Postdoctoral Fellow in Differential Geometry. Andrea received his bachelor's degree from Torino University in 2006 and his master's degree from SISSA & Trieste University in 2008. In September 2011, he obtained his Ph.D. from SISSA (Trieste) under the supervision of Andrea Malchiodi. Starting in September 2016, he will be an assistant professor at the University of Warwick. Andrea is broadly interested in differential geometry/geometric analysis and its application to general relativity and the optimal shapes of cell membranes. In his work on variational geometric problems, he has investigated the optimal shapes that arise as minimizers — or more generally as stationary points — of energy functionals. Andrea is also interested in non-smooth spaces that satisfy lower Ricci curvature bounds in the optimal transportation sense. With collaborators, he proved results about the structure of such spaces and about their geometric-analytic properties.

The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.

Focus on the Scientist: Tobias Colding

Tobias Colding, a Clay Senior Scholar and Eisenbud Professor in this semester's Differential Geometry program, is no stranger to the pleasures of MSRI — he also held a Clay Senior Scholar position here in 2011 as part of the Quantitative Geometry program. He is also one of the organizers of the current Differential Geometry program.

Tobias Colding

Tobias earned his Ph.D. from the University of Pennsylvania in 1992 and completed his final year of graduate studies while visiting MSRI with his advisor Chris Croke. During his time as a Courant Instructor at NYU, he returned to MSRI as a postdoctoral fellow in 1993 as part of that year's Differential Geometry program. He rose through the ranks at NYU and was named a full Professor in 1999, a position he maintained until 2008. After holding visiting positions at MIT and Princeton, he moved permanently to MIT in 2005, where he is now the Cecil and Ida Green Distinguished Professor.

Tobias is undaunted by mathematical challenges since his enthusiasm and optimism propel him. While to many mathematicians, Tobias is best known for his work on the structure of spaces with lower Ricci curvature bounds, we focus on two of his most recent seminal contributions in the calculus of variations.

In 2010, Tobias was awarded the Oswald Veblen prize, jointly with William P. Minicozzi, II, for their "profound work on minimal surfaces." In a series of four papers published in 2004, the pair determined a compactness theory for sequences of minimal disks without presuming curvature or area bounds. This result was immediately applied to resolve many open problems including the longstanding conjecture on the uniqueness of the helicoid.

Their work began with the seemingly basic — but highly technical — observation that every embedded minimal disk can be locally modeled by a minimal graph or a piece of the helicoid. With the help of this local picture, they determined global structural results for embedded minimal surfaces with finite genus. In particular, they resolved the Calabi–Yau conjecture for minimal surfaces with finite topology, proving that every complete, embedded example must be proper.

Again in collaboration with Minicozzi, Tobias recently developed infinite-dimensional Łojasiewicz inequalities for noncompact hypersurfaces with bounded entropy. The noncompactness required new techniques because, unlike in Leon Simon's celebrated work, they cannot prove these inequalities by reducing from an infinite-dimensional space to the classical setting. As an immediate application, they established the uniqueness of tangent flows at a generic singularity of the mean curvature flow, a result which itself has many further applications.

Tobias is a member of the American Academy of Arts and Sciences and the Royal Danish Academy of Sciences and Letters. He was also an invited speaker to the International Congress of Mathematicians in Berlin in 1998.

— Christine Breiner

Named Positions for Spring 2016

MSRI is grateful for the generous support that comes from endowments, annual gifts, and awards that support faculty and postdoc members of its programs each semester.

Eisenbud and Simons Professors

Olivier Biquard, École Normale Supérieure
Simon Brendle, Stanford University
Sun-Yung Chang, Princeton University
Tobias Colding, Massachusetts Institute of Technology
Simon Donaldson, Imperial College, London
Akito Futaki, The University of Tokyo
Ursula Hamenstaedt, Universitäat Bonn
Tom Ilmanen, ETH Zurich
Vitali Kapovitch, University of Toronto
Zhiqin Lu, University of California, Irvine
Curtis McMullen, Harvard University
Richard Schoen, Stanford University
Felix Schulze, University College London
Natasa Sesum, Rutgers University
Michael Singer, University College London
Gang Tian, Princeton University
Jeff Viaclovsky, University of Wisconsin
Guofang Wei, University of California, Santa Barbara
Xiaohua Zhu, Peking University

Named Postdoctoral Fellows

Huneke: Andrea Mondino, ETH Zurich
Strauch: Heather Macbeth, Massachusetts Institute of Technology
Viterbi: Chengjian Yao, Stony Brook University

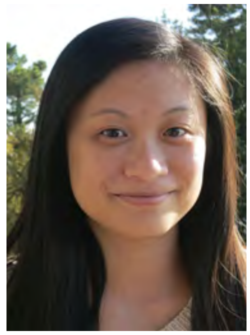
Clay Senior Scholar

Tobias Colding, Massachusetts Institute of Technology

Staff News: Recognizing Five New MSRI Faces . . .


Patricia Brody


Aaron Hale


Tracy Huang


Jennifer Murawski


Britta Shillingsburg

Patricia Brody joined MSRI in December as the new Housing Advisor. Prior to coming to MSRI, Patricia ran a nonprofit foundation in San Francisco. She graduated from UCLA with a B.A. in political science and earned her J.D. there as well, and has been active in the legal world. Her favorite pastimes include time with her family and travel — and housing!

Aaron Hale joined MSRI in January as IT Strategist and Technical Lead. Aaron has worked as an IT contractor and consultant for four years. Prior to that, Aaron worked as a lab coordinator at Pennsylvania College of Technology. In his free time, Aaron likes to spend time with his wife and daughter, camping and hiking around the Bay Area. His interests include particle physics, foundational problems in quantum mechanics, and theoretical biology.

Tracy Huang joined MSRI in January as the Assistant for Scientific Activities. Tracy recently graduated from the University of Southern California with a B.A. in East Asian languages and cultures. She has studied in Beijing and Nanjing, China, and she is looking forward to more travel in her future.

Jennifer Murawski joined MSRI in December as the Communications and Event Coordinator. She was previously an assistant director at the University of Pittsburgh's Asian Studies Center, coordinating undergraduate academic programs and K-12 educational outreach. She is an alumna of the Japan Exchange and Teaching Program and volunteers with US–Japan sister cities to promote youth exchange programs.

Britta Shillingsburg joined MSRI in December as Event Planner and Marketing Specialist, based out of Washington, DC. She previously worked for the Direct Selling Association as an exhibit and marketing coordinator. Britta volunteers with the DC Lean In Executive Board to plan professional development events for women, and she teaches high school students about small business ownership for Pennsylvania Free Enterprise Week. Her hobbies include yoga, hiking, and running.

Jackie Blue in the Berlekamp Garden

. . . and Honoring 32 Years of Service

In January, MSRI directors and staff gathered to say farewell to **Jackie Blue** on the occasion of her retirement. Jackie was MSRI's International Scholars and Housing Advisor for the last 32 years, helping our visiting scholars and their families find accommodation and obtain the proper visas, and providing judicious and friendly advice. In honor of her many years of service, Jackie's name will be added to the commemorative wall in MSRI's outdoor Berlekamp Garden.

Clay Senior Scholarships

The Clay Mathematics Institute (www.claymath.org) has announced the 2016–2017 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in a topical program at an institute or university away from their home institution. Here are the Clay Senior Scholars who will work at MSRI in 2016–2017:

Geometric Group Theory (Fall 2016)
Karen Vogtmann, University of Warwick

Harmonic Analysis (Spring 2017)
Alexander Volberg, Michigan State University

Analytic Number Theory (Spring 2017)
Manjul Bhargava, Princeton University
Rodney Heath-Brown, Oxford University

CME Group–MSRI Prize in Innovative Quantitative Applications

Douglas W. Diamond

The tenth annual CME Group–MSRI Prize in Innovative Quantitative Applications was awarded to Douglas W. Diamond, Merton H. Miller Distinguished Service Professor of Finance at the University of Chicago Booth School of Business, at a luncheon in Chicago in February.

Diamond's research has focused on the study of financial intermediaries, financial crises, and liquidity. A panel discussion on "Non-Bank Runs and Financial Crises" was held in conjunction with the award ceremony.

The annual CME Group–MSRI Prize is awarded to an individual or a group to recognize originality and innovation in the use of mathematical, statistical, or computational methods for the study of the behavior of markets, and more broadly of economics. You can read more about the CME Group–MSRI Prize at http://tinyurl.com/cme-msri.

Forthcoming Workshops

May 2–6, 2016: *Geometric Flows in Riemannian and Complex Geometry*, organized by Tobias Colding, John Lott (Lead), Natasa Sesum

June 11–July 24, 2016: *MSRI-UP 2016: Sandpile Groups*, organized by Federico Ardila, Duane Cooper, Maria Mercedes Franco, Herbert Medina, Suzanne Weekes (Lead)

August 17–19, 2016: *Connections for Women: Geometric Group Theory*, organized by Ruth Charney (Lead), Indira Chatterji, Mark Feighn, Talia Fernós

August 22–26, 2016: *Introductory Workshop: Geometric Group Theory*, organized by Martin Bridson, Benson Farb, Zlil Sela (Lead), Karen Vogtmann

September 27–30, 2016: *Groups Acting on* $\mathrm{CAT}(0)$ *Spaces*, organized by Ian Agol, Pierre-Emmanuel Caprace, Koji Fujiwara, Alessandra Iozzi, Michah Sageev (Lead)

October 25–28, 2016: *Geometry of Mapping Class Groups and* $\mathrm{Out}(F_n)$, organized by Yael Algom-Kfir, Mladen Bestvina (Lead), Richard Canary, Gilbert Levitt

December 6–9, 2016: *Amenability, Coarse Embeddability and Fixed Point Properties*, organized by Goulnara Arzhantseva, Cornelia Drutu (Lead), Graham Niblo, Piotr Nowak

Summer Graduate Schools

May 30–June 11, 2016: *Séminaire de Mathématiques Supérieures 2016: Dynamics of Biological Systems (Canada)*, organized by Thomas Hillen, Mark Lewis, Yingfei Yi

June 13–June 24, 2016: *Harmonic Analysis and Elliptic Equations on Real Euclidean Spaces and on Rough Sets*, organized by Steven Hofmann (Lead), Jose Maria Martell

June 20–July 1, 2016: *Mixed Integer Nonlinear Programming: Theory, Algorithms and Applications (Spain)*, organized by Francisco Castro-Jiménez, Elena Fernández, Justo Puerto

July 11–22, 2016: *An Introduction to Character Theory and the McKay Conjecture*, organized by Robert Guralnick, Pham Tiep

July 18–29, 2016: *Electronic Structure Theory*, organized by Lin Lin (Lead), Jianfeng Lu, James Sethian

July 25–August 5, 2016: *Chip Firing and Tropical Curves*, organized by Matthew Baker (Lead), David Jensen, Sam Payne

For more information about any of these workshops, as well as a full list of all upcoming workshops and programs, please see www.msri.org/scientific.

Call for Membership

MSRI invites membership applications for the 2017–2018 academic year in these positions:

Research Professors	by October 1, 2016
Research Members	by December 1, 2016
Postdoctoral Fellows	by December 1, 2016

In the academic year 2017–2018, the research programs are:

Geometric Functional Analysis and Applications
Aug 14–Dec 15, 2017
Organized by Franck Barthe, Marianna Csornyei, Boaz Klartag, Alexander Koldobsky, Rafal Latala, Mark Rudelson

Geometric and Topological Combinatorics
Aug 14–Dec 15, 2017
Organized by Jesus De Loera, Vic Reiner, Francisco Santos, Francis Su, Rekha Thomas, Günter M. Ziegler

Group Representation Theory and Applications
Jan 16–May 25, 2018
Organized by Robert Guralnick, Alexander (Sasha) Kleshchev, Gunter Malle, Gabriel Navarro, Julia Pevtsova, Raphael Rouquier, Pham Tiep

Enumerative Geometry Beyond Numbers
Jan 16–May 25, 2018
Organized by Mina Aganagic, Denis Auroux, Jim Bryan, Andrei Okounkov, Balazs Szendroi

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at www.mathjobs.org after August 1, 2016. For more information about any of the programs, please see www.msri.org/scientific/programs.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

1. As a respite from the current election discussions, it seems useful to recall and extend a problem (from many years ago) with the hope of elevating conversations about elections to a more rarefied level.

You are a citizen of Oddonia, which has two political parties. The parties are so closely matched that from your point of view anyone else's vote might as well be viewed as determined by the flip of a fair coin.

Oddonia has three equally populous states, each with $2n+1$ voters. Two systems for electing the President are being considered: (1) a popular vote, or (2) an electoral college with one electoral vote for each state.

(a) Which system gives the most power to you? You should regard one system as more powerful than another if the probability that your vote is decisive (that is, determines the election) is larger.

(b) More generally, what can you say about comparing electoral college systems assuming d or d' states, where d and d' are divisors of the total population size?

2. Four people come to a river in the night. There is a rickety narrow bridge, and it can only hold at most two of the people at a time. Moreover, it is very dark and there is no railing, so a flashlight must be used whenever the bridge is crossed. There is only one flashlight.

Person A can cross the bridge in one minute, B in two minutes, C in five minutes, and D in eight minutes. When two people cross the bridge together, they must of course move at the slower person's pace. How quickly can they all get across the bridge?

Comment: This came up in conversation recently, and we learned that the problem is "well-known," at least in the sense that it has its own wiki page.

3. For $n > 1$, consider an $n \times n$ chessboard and place identical pieces at the centers of different squares.

(a) Show that no matter how $2n$ identical pieces are placed on the board, one can always find four pieces among them that are the vertices of a parallelogram.

(b) Show that there is a way to place $(2n-1)$ identical chess pieces so that no four of them are the vertices of a parallelogram.

4. Find a positive integer N and $a_1, a_2, \ldots, a_N$, where each a_i is equal to 1 or -1, such that

$$20162016 = a_1 \cdot 1^3 + a_2 \cdot 2^3 + a_3 \cdot 3^3 + \cdots + a_N \cdot N^3,$$

or show that this is impossible.

Comment: The preceding two problems (3 and 4) were both on the 2016 Bay Area Mathematical Olympiad, which had a number of excellent problems. Problem 6 below was somehow inspired by problem 4; although both succumb to a specific technique, there seem to be a larger number of ways to approach problem 6.

5. Alice, Bob, and Catherine have positive integers on their foreheads. One of the numbers is the sum of the other two. (Each can see the other two but not their own, no side communication is allowed, each knows that one number is the sum of the other two, they are all brilliant logicians, etc.).

The following dialog ensues:

Alice:	I don't know my number.
Bob :	I don't know my number.
Catherine :	I don't know my number.
Alice :	I know my number, and it is 50.

What are the other two numbers?

Comment: This problem apparently first appeared in a 2004 issue of *Math Horizons*; we ran into it in the new Springer book *One Hundred Prisoners and a Light Bulb* by Ditmarsch and Kool, which contains a number of such brainteasers as well as an overview of epistemic logic.

6. Show that every sufficiently large integer is a sum of cubes of distinct positive integers.

Mathematical Art at MSRI

Henry Segerman (far left) brought an exhibit of 3D-printed "illustrated mathematics" to display at MSRI in February 2016. The display featured twelve pieces of his solo and collaborative artwork, which can also be viewed online at www.segerman.org.

Dr. Segerman is an assistant professor in the Department of Mathematics at Oklahoma State University, working mostly in three-dimensional geometry and topology.

2015 Annual Report

We gratefully acknowledge the supporters of MSRI whose generosity allows us to fulfill MSRI's mission to advance and communicate the fundamental knowledge in mathematics and the mathematical sciences; to develop human capital for the growth and use of such knowledge; and to cultivate in the larger society awareness and appreciation of the beauty, power, and importance of mathematical ideas and ways of understanding the world.

This report acknowledges grants and gifts received from January 1 – December 31, 2015. In preparation of this report, we have tried to avoid errors and omissions. If any are found, please accept our apologies, and report them to development@msri.org. If your name was not listed as you prefer, please let us know so we can correct our records. If your gift was received after December 31, 2015, your name will appear in the 2016 Annual Report. For more information on our giving program, please visit www.msri.org.

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We recognize our most generous and loyal supporters whose leadership and commitment ensures MSRI continues to thrive as one of the world's leading mathematics research centers.

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Andrew Viterbi

Museion Society

The Museion Society, named after Musaeum, the Hall of the Muses in ancient Alexandria, recognizes our leadership donors in annual and endowment giving.

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Archimedes Society

The Archimedes Society recognizes MSRI's annual supporters.

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