

**Final Report
on the
Mathematical Sciences Research Institute
2015–16 Activities
supported by
NSA Grant H98230-15-1-0106
Support of Early Career Researchers at MSRI
8/01/2015-5/31/2016**

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**Mathematical Sciences Research Institute
NSA Final Report for H98230-15-1-0106**

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I. INTRODUCTION

The main scientific activities of MSRI are its Programs and Workshops. MSRI hosts two to four semester-long programs each year. Each program has about forty mathematicians in residence at any given time, including seven to eight graduate students.

Generally, each semester-long program features three workshops. A program begins with a *Connections for Women Workshop*, which is designed to encourage the participation of women in the research activities of the program. If the area of mathematics is one that traditionally has a large number of women, then the workshop can be used to highlight and to showcase their individual work. However, if the number of women in the field is low, then the workshop is marketed to a wider female audience in an effort to stimulate interest in the area. In addition, another goal is to encourage new connections among the women early in the program as a catalyst for eventual collaborations. This workshop is then followed by an *Introductory Workshop*, the purpose of which is to introduce the subject to the broader mathematical community. Later during the program, there is a *Topical Workshop*, which is designed to explore some of the themes of the program in greater depth.

In addition to the scientific workshops that run parallel with the programs, MSRI hosts a Hot Topics workshop. These workshops are intended to explore emerging topics in mathematics. (See Section II.B for a brief summary)

MSRI also hosts and co-hosts Education & Outreach Workshops. These workshops focus on improving the skills of K–12 math teachers. Their descriptions, as well as lists of speakers, talks and participants, can be found on the MSRI web site at <http://www.msri.org/web/msri/education/for-k-12-educators>.

Another essential activity at MSRI is its series of Summer Graduate Schools which target advanced graduate students in mathematics. During the summer of 2015, MSRI hosted three on-site and three off-site Summer Graduate Schools, with themes ranging from algebraic topology topics to geometry and analysis. A complete description can be found at the URL <http://www.msri.org/web/msri/scientific/workshops/summer-graduate-school>.

Last but not least, each summer since 2007, MSRI has hosted a summer school (MSRI-UP) for undergraduate students with the aim of increasing the number of PhDs among members of under-represented groups. These summer schools are co-funded by the NSA and the NSF. The 2015 MSRI-UP, *Geometric Combinatorics Motivated by the Social Sciences*, was a successful and popular school, with 17 undergraduate participants. (See Section II.B, for a brief summary). Since MSRI-UP is funded by an independent NSA grant, its report is filed separately.

II. OVERVIEW OF ACTIVITIES 2015–16

During the 2015–16 academic year, MSRI hosted two jumbo programs: *New Challenges in PDE: Deterministic Dynamics and Infinite Dimensional Systems* took place in the fall of 2015 and *Differential Geometry* took place in the spring of 2016. MSRI also hosted a small Complementary Program for mathematicians whose interests are not closely related to the two jumbo programs. They are briefly summarized in Section II. A

There were 242 researchers who participated in these programs for a period of one month or longer. Of those members, there were 35 Postdoctoral Fellows, 47 Research Professors, 123 Research Members, and 37 Program Associates (Graduate Students).

The NSA grant H98230-15-1-0106 funded four Postdoctoral Fellows: Boaz Haberman and Nathan Totz participated in the fall program, *New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems*. Jeremy Leach and Michael Lock participated in the spring program, *Differential Geometry*.

A. Major Programs and their Associated Workshops

Note: The description of each activity is provided to MSRI by the organizers prior to the beginning of each activity; therefore, the verbs are in future tense. In the list of organizers of each activity, an asterisk (*) denotes lead organizer(s).

Program 1: New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems

August 17, 2015 - December 18, 2015

*Organizers: Kay Kirkpatrick (University of Illinois at Urbana-Champaign), Yvan Martel (École Polytechnique), Jonathan Mattingly (Duke University), Andrea Nahmod (University of Massachusetts, Amherst), Pierre Raphael (Université Nice Sophia-Antipolis), Luc Rey-Bellet (University of Massachusetts, Amherst), *Gigliola Staffilani (Massachusetts Institute of Technology), Daniel Tataru (University of California, Berkeley)*

The fundamental aim of this program is to bring together a core group of mathematicians from the general communities of nonlinear dispersive and stochastic partial differential equations whose research contains an underlying and unifying problem: quantitatively analyzing the dynamics of solutions arising from the flows generated by deterministic and non-deterministic evolution differential equations, or dynamical evolution of large physical systems, and in various regimes.

In recent years there has been spectacular progress within both communities in the understanding of this common problem. The main efforts exercised have generated an incredible number of deep results that are not just beautiful mathematically, but are also important to understand the complex natural phenomena around us. Yet, many open questions and challenges remain ahead of us. Hosting the proposed program at MSRI

would be the most effective venue to explore the specific questions at the core of the unifying theme and to have a focused and open exchange of ideas, connections and mathematical tools leading to potential new paradigms. This special program will undoubtedly produce new and fundamental results in both areas, and possibly be the start of a new generation of researchers comfortable on both languages.

Workshops associated with the *New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems* program:

Workshop 1: Connections for Women: Dispersive and Stochastic PDE

August 19, 2015 - August 21, 2015

*Organizers: *Kay Kirkpatrick (University of Illinois at Urbana-Champaign), Andrea Nahmod (University of Massachusetts, Amherst)*

This workshop will consist of various talks given by prominent female mathematicians whose research lies in and interfaces with the fields of nonlinear evolution dispersive PDE, wave phenomena and stochastic processes. These talks will be appropriate for graduate students, post-docs, and researchers in areas above mentioned. The workshop will allocate ample time for group discussions and will include a professional development session.

Workshop 2: Introductory Workshop: Randomness and long time dynamics in nonlinear evolution differential equations

August 24, 2015 - August 28, 2015

*Organizers: Kay Kirkpatrick (University of Illinois at Urbana-Champaign), *Yvan Martel (École Polytechnique), *Luc Rey-Bellet (University of Massachusetts, Amherst), Gigliola Staffilani (Massachusetts Institute of Technology)*

The purpose of the program *New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems* is to bring together a core group of mathematicians from the dispersive PDE and the SPDE communities whose research contains an underlying and unifying problem: analyzing high or infinite dimensional dynamics, where dynamics is understood in a broad sense and arising from the flows generated by either deterministic or stochastic partial differential equations, or from dynamical evolution of large physical systems.

The introductory workshop will serve as an overview to the program. It aims at familiarizing graduate students, postdocs, and other researchers to the major topics of the program through short courses and discussions.

Workshop 3: New challenges in PDE: Deterministic dynamics and randomness in high and infinite dimensional systems

October 19, 2015 - October 30, 2015

*Organizers: Jonathan Mattingly (Duke University), *Andrea Nahmod (University of Massachusetts, Amherst), Pierre Raphael (Université Nice Sophia-Antipolis), Luc Rey-Bellet (University of Massachusetts, Amherst), Daniel Tataru (University of California, Berkeley)*

This workshop serves to bring into focus the fundamental aim of the jumbo program by both a) showcasing the spectacular progress in recent years in the study of both nonlinear dispersive as well as stochastic partial differential equations and b) bringing to the fore the key challenges for the future in quantitatively analyzing the dynamics of solutions arising from the flows generated by deterministic and non-deterministic evolution differential equations, or dynamical evolution of large physical systems.

During the two weeks long workshop, we intertwine talks on a wide array of topics by some of the key researchers in both communities and aim at highlighting the most salient ideas, proofs and questions which are important and fertile for 'cross-pollination' between PDE and SPDE. Topics include: Global dynamics and singularity formation for geometric and physical nonlinear wave and dispersive models (critical and supercritical regimes); dynamics of infinite dimensional systems (critical phenomena, multi scale dynamics and metastability); symplectic structures of infinite dimensional dynamical systems; randomization and long time dynamics, invariant Gibbs and weighted Wiener measures; derivation of effective dynamics in quantum systems; weak turbulence phenomena; optimization and learning algorithms: distributed, stochastic and parallel.

Program 2: Differential Geometry

January 11, 2016 – May 20, 2016

*Organizers: Tobias Colding (Massachusetts Institute of Technology), Simon Donaldson (Imperial College, London), John Lott (University of California, Berkeley), Natasa Sesum (Rutgers University), Gang Tian (Princeton University), *Jeff Viaclovsky (University of Wisconsin-Madison)*

Differential geometry is a subject with both deep roots and recent advances. Many old problems in the field have recently been solved, such as the Poincaré and geometrization conjectures by Perelman, the quarter pinching conjecture by Brendle-Schoen, the Lawson Conjecture by Brendle, and the Willmore Conjecture by Marques-Neves. The solutions of these problems have introduced a wealth of new techniques into the field. This semester-long program will focus on the following main themes:

- (1) Einstein metrics and generalizations,
- (2) Complex differential geometry,
- (3) Spaces with curvature bounded from below,
- (4) Geometric flows, and particularly on the deep connections between these areas.

Workshops associated with the *Differential Geometry* program:

Workshop 1: Connections for Women: Differential Geometry

January 14, 2016 – January 15, 2016

*Christine Breiner (Fordham University), *Natasa Sesum (Rutgers University)*

The purpose of this meeting is to help junior female researchers to become familiar with the focus topics of the main MSRI program, and also for the junior researchers to have an opportunity to get acquainted with more senior women researchers in differential geometry. This workshop is open to *all* mathematicians.

Workshop 2: Introductory Workshop: Modern Riemannian Geometry

January 18, 2016 – January 22, 2016

**Tobias Colding (Massachusetts Institute of Technology), John Lott (University of California, Berkeley), Jeff Viaclovsky (University of Wisconsin-Madison)*

The week will be devoted to an introduction to modern techniques in Riemannian geometry. This is intended to help graduate students and younger researchers get a head start, in order to increase their participation during the main semester programs and research lectures. To increase outreach, the week will focus on Riemannian geometry and should be largely accessible. Some minicourses on topics of recent interest will be included. The workshop will also have semi-expository lectures dealing with aspects of spaces with curvature bounded from below, since such spaces will occur throughout the semester. We expect that many Berkeley mathematicians and students will participate in the introductory workshop.

Workshop 3: Kähler Geometry, Einstein Metrics, and Generalizations

March 21, 2016 – March 25, 2016

*Olivier Biquard (École Normale Supérieure), Simon Donaldson (Imperial College, London), Gang Tian (Princeton University), *Jeff Viaclovsky (University of Wisconsin-Madison)*

The workshop will integrate elements from complex differential geometry with Einstein metrics and their generalizations. The topics will include

- Existence of Kähler-Einstein metrics and extremal Kähler metrics. Notions of stability in algebraic geometry such as Chow stability, K-stability, b-stability, and polytope stability. Kähler-Einstein metrics with conical singularities along a divisor.
- Calabi-Yau metrics and collapsed limit spaces. Connections with physics and mirror symmetry.
- Einstein metrics and their moduli spaces, ϵ -regularity, noncompact examples such as ALE, ALF, and Poincaré -Einstein metrics. Generalizations of the Einstein condition, such as Bach-flat metrics and Ricci solitons.
- Sasaki-Einstein metrics and metrics with special holonomy. New examples and classification problems

Organized in partnership with Clay Mathematics Institute.

Workshop 4: Geometric Flows in Riemannian and Complex Geometry

May 02, 2016 – May 06, 2016

*Tobias Colding (Massachusetts Institute of Technology), *John Lott (University of California, Berkeley), Natasa Sesum (Rutgers University)*

The workshop will concentrate on parabolic methods in both Riemannian and complex geometry. The topics will include

- Ricci flow. Analytic questions about Ricci flow in three dimensions. Possible applications of Ricci flow to 4-manifold topology. Ricci flow in higher dimensions under curvature assumptions.
- Kähler-Ricci Flow. Applications to the Kähler-Einstein problem. Connections to the minimal model program. Study of Kähler-Ricci solitons and limits of Kähler-Ricci flow.
- Mean curvature flow. Singularity analysis. Generic mean curvature flow.
- Other geometric flows such as Calabi flow and pluriclosed flow.

Organized in partnership with Clay Mathematics Institute.

Program 3: Complementary Program (2015-16)

The Complementary Program has a limited number of memberships that are open to mathematicians whose interests are not closely related to the core programs; special consideration is given to mathematicians who are partners of an invited member of a core program.

B. Hot Topics Workshop

Hot Topics: Cluster algebras and wall crossing

March 28, 2016 - April 01, 2016

*Organizers: *Mark Gross (University of Cambridge), Paul Hacking (University of Massachusetts, Amherst), Sean Keel (University of Texas), Lauren Williams (University of California, Berkeley)*

Cluster algebras were introduced in 2001 by Fomin and Zelevinsky to capture the combinatorics of canonical bases and total positivity in semisimple Lie groups. Since then they have revealed a rich combinatorial and group-theoretic structure, and have had significant impact beyond these initial subjects, including string theory, algebraic geometry, and mirror symmetry. Recently Gross, Hacking, Keel and Kontsevich released a preprint introducing mirror symmetry techniques into the subject which resolved several long-standing conjectures, including the construction of canonical bases for cluster algebras and positivity of the Laurent phenomenon. This preprint reformulates the basic construction of cluster algebras in terms of scattering diagrams (or wall-crossing structures). This leads to the proofs of the conjectures and to new constructions of elements of cluster algebras. But fundamentally they provide a new tool for thinking about cluster algebras.

The workshop will bring together many of the different users of cluster algebras to

achieve a synthesis of these new techniques with many of the different aspects of the subject. There will be lecture series on the new techniques, and other lecture series on connections with Lie theory, quiver representation theory, mirror symmetry, string theory, and stability conditions.

III. PARTICIPATION SUMMARY

A. All MSRI Members

The table on the next two pages indicates the number of participants for each activity that took place at MSRI during the 2015–16 academic year.

Time	Activity Type	Activity Title	No. of participants
Fall 2015	Scientific Program	New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems	109
August 19, 2015 - August 21, 2015	Programmatic Workshop	Connections for Women: Dispersive and Stochastic PDE	62
August 24, 2015 - August 28, 2015	Programmatic Workshop	Introductory Workshop: Randomness and long time dynamics in nonlinear evolution differential equations	111
October 19, 2015 - October 30, 2015	Programmatic Workshop	New challenges in PDE: Deterministic dynamics and randomness in high and infinite dimensional systems	156
Spring 2016	Scientific Program	Differential Geometry	122
January 14, 2016 - January 15, 2016	Programmatic Workshop	Connections for Women: Differential Geometry	49
January 18, 2016 - January 22, 2016	Programmatic Workshop	Introductory Workshop: Modern Riemannian Geometry	128
March 21, 2016 - March 25, 2016	Programmatic Workshop	Kähler Geometry, Einstein Metrics, and Generalizations	134
May 02, 2016 - May 06, 2016	Programmatic Workshop	Geometric Flows in Riemannian and Complex Geometry	135
Academic Year 2015-16	Scientific Program	Complementary Program 2015-16	11

Time	Activity Type	Activity Title	No. of participants
June 13, 2015 – July 26, 2015	MSRI-UP 2015	Geometric Combinatorics Motivated by the Social Sciences	18
June 15, 2015 - June 26, 2015	Summer Graduate School (2015)	Seminaire de Mathematiques Superieures 2015: Geometric and Computational Spectral Theory	18
June 15, 2015 - July 11, 2015	Summer Graduate School (2015)	CRM-PIMS Summer School in Probability	18
June 15, 2015 - June 26, 2015	Summer Graduate School (2015)	Geometric Group Theory	59
June 29, 2015 - July 10, 2015	Summer Graduate School (2015)	NIMS Summer School on Random Matrix Theory	15
June 29, 2015 - July 10, 2015	Summer Graduate School (2015)	Mathematical Topics in Systems Biology	25
July 6, 2015 - July 17, 2015	Summer Graduate School (2015)	Berkeley summer course in mining and modeling of neuroscience data	6
July 13, 2015 – July 24, 2015	Summer Graduate School (2015)	Gaps between Primes and Analytic Number Theory	47
July 27, 2015 – August 7, 2015	Summer Graduate School (2015)	Incompressible Fluid Flows at High Reynolds Number	51
December 5, 2015	Other Scientific Workshop	Bay Area Differential Geometry Seminar (BADGS) Winter 2015	30
April 30, 2016	Other Scientific Workshop	Bay Area Differential Geometry Seminar (BADGS) Spring 2016	30
March 28, 2016 - April 01, 2016	Other Scientific Workshop	Hot Topics: Cluster algebras and wall-crossing	73
February 10, 2016 - February 12, 2016	Education & Outreach Workshop	Critical Issues in Mathematics Education 2016: Observing, Evaluating and Improving Mathematics Teaching from the Early Grades through the University	112

B. NSA supported Postdoctoral Fellows

There were 35 postdoctoral fellows who participated in the 2015–16 programs. Four of these fellows were self-supported and 31 were funded through MSRI’s NSF and NSA grants. The NSA grant, H98230-15-1-0106, funded four out of the 31 postdoctoral fellows.

Boaz Haberman participated in the fall 2015 program, *New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems*



Haberman, Boaz

Name: Boaz Haberman
Year of Ph.D: 2015
Institution of Ph.D.: UC Berkeley
Dissertation title: Inverse problems with rough data
Ph.D. advisor: Daniel Tataru

Mentor while at MSRI: Herbert Koch

Institution prior to obtaining the MSRI PD fellowship: UC Berkeley
Position at that institution: Graduate student
Mentor (if applicable): Daniel Tataru

Institution (or company) where you are going after the MSRI PD fellowship: University of Chicago
Position: L.E. Dickson Instructor
Anticipated length: 3 years
Mentor (if applicable): Carlos Kenig

Boaz Haberman's comments:

While I was at MSRI I worked on finishing a result that I had partially proven in my thesis. I showed that a Schrodinger operator $D^2 + A.D + D.A + q$ on a bounded domain in \mathbb{R}^3 is determined by its Dirichlet-to-Neumann map, assuming that A is small in $W^{\{0+\epsilon, 3\}}$ and q is only in $W^{\{-1, 3\}}$. I recently posted a preprint at arXiv:1512.01580. I am working on improving this to treat the critical case where A is large in L^3 .

While at MSRI I had the opportunity to discuss my work with Daniel Tataru and Herbert Koch, which was very useful in order to make progress. I have now finally figured out a key estimate, and have a program to treat the critical case, which requires a much more precise analysis than the subcritical case.

Was your experience at MSRI beneficial? Why or why not?

My experience at MSRI was very beneficial. So far my work has been mainly in one direction, and being at MSRI exposed me to many interesting problems. I feel that I benefited from the consistent theme of the semester as well as from the diversity of subjects presented. I also benefited from talking to participants about their research and gaining new perspectives.

Nathan Totz participated in the fall 2015 program, *New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems*



Totz, Nathan

Name: Nathan Totz
Year of Ph.D: 2011
Institution of Ph.D.: University of Michigan
Dissertation title: A Rigorous Justification of the Modulation Approximation to the 2D Full Water Wave Problem
Ph.D. advisor: Sijue Wu

Mentor while at MSRI: Natasa Pavlovic

Institution prior to obtaining the MSRI PD fellowship: University of Massachusetts Amherst
Position at that institution: Visiting Assistant Professor
Mentor (if applicable): Andrea Nahmod

Institution (or company) where you are going after the MSRI PD fellowship: University of Massachusetts Amherst
Anticipated length: 2 years
Mentor (if applicable): Andrea Nahmod

Nathan Totz's comments:

I finished and submitted the paper "Global Well-Posedness of 2D Non-Focusing Schrodinger Equations via Rigorous Modulation Justification

Was your experience at MSRI beneficial? Why or why not?

Definitely! My research was rapidly sped up thanks to the intense environment, the other very knowledgeable participants, and the time to focus.

Jeremy Leach participated in the spring 2016 program, *Differential Geometry*



Leach, Jeremy

Name: Jeremy Leach
Year of Ph.D.: 2015
Institution of Ph.D.: Stanford University
Dissertation title: The vacuum Einstein constraint equations on manifolds with ends of cylindrical type
Ph.D. advisor: Rafe Mazzeo
Mentor while at MSRI: Robert Kusner

Institution prior to obtaining the MSRI PD fellowship: University of Washington
Position at that institution: Acting Assistant Professor
Mentor (if applicable): Daniel Pollack

Institution (or company) where you are going after the MSRI PD fellowship: Trexquant Investment LP
Position: Portfolio Manager
Anticipated length: Permanent

Jeremy Leach's comments:

I spent the first three weeks of the program finishing up a paper (pertaining to the Einstein constraint equations on manifolds with both asymptotically Euclidean and asymptotically conformally cylindrical ends) which I had started working on while I was a postdoc at the University of Washington.

After submitting that paper to *Classical and Quantum Gravity*, I split the remainder of my time at MSRI between two projects. The first was an independent effort to analyze the sign of the energy density associated to the Alcubierre metric in the conformal gravity model. I have made progress, thanks in large part to the access to Mathematica which MSRI granted me. Though this project is still ongoing, I expect this work to result in a published paper in the near future.

The other project is a joint work with both my MSRI mentor, Robert Kusner, and my Ph.D. advisor, Rafe Mazzeo. Early in the semester, my advisor proposed the following question to me: Is the cylinder the only conformally flat Riemannian manifold with constant positive scalar curvature and a finite number of asymptotically cylindrical ends? There is an analogue to this question in the mean curvature setting as well. We currently believe that we can construct a counterexample which suggests that the answer to this question is "no", though there are still gaps in the proof which we believe we can fix. If we are successful, this project will also certainly result in a published paper.

Jeremy Leach comments, continued...

Was your experience at MSRI beneficial? Why or why not?

My time at MSRI was extremely beneficial. Though it is true that my academic research has no bearing on my future in industry, my semester at MSRI has given me the opportunity to do precisely what drew me to mathematics in the first place: research and collaboration. The ability to dedicate all of my time to these pursuits has made my semester here the most enjoyable and rewarding mathematical experience of my life, and much of the impact I will have on my field of research will be owed to

Michael Lock participated in the spring 2016 program, *Differential Geometry*



Lock, Michael

Name: Michael Lock

Year of Ph.D: 2013

Institution of Ph.D.: University of Wisconsin - Madison

Dissertation title: Index theorems for anti-self-dual and self-dual orbifolds

Ph.D. advisor: advisor: Jeff Viaclovsky

Mentor while at MSRI: Robert Kusner

Institution prior to obtaining the MSRI PD fellowship: University of Texas at Austin

Position at that institution: RTG Postdoctoral Fellow Mentor

Mentor (if applicable): Dan Freed

Institution (or company) where you are going after the MSRI PD fellowship: Leaving academia

Michael Lock's comments:

While at MSRI I worked on problems concerning special Hermitian metrics in the non-Kähler setting. I completed the work for one publication that is forthcoming, as well as answered questions regarding a generalization of previous work that will be in a forthcoming publication as well.

Was your experience at MSRI beneficial? Why or why not?

My experience at MSRI was very beneficial because it broadened my research interests. I'm actually leaving academia after this semester. I made the decision to at the end of last summer though, so it has nothing to do with my experience at MSRI which was wonderful. I still enjoy my work in math very much, and benefited from the experience at MSRI, I just have had a growing desire to try something completely new which is why I'm leaving math.

VI. PUBLICATIONS SUMMARY

The four NSA funded postdoctoral fellows worked on a total of four papers during their stay at MSRI.

Member Name	Paper Titles	Co-author(s)	Paper Status
Michael Lock	Existence and uniqueness of scalar difference metrics	Michael G. Dabkowski	Rough/Draft
Jeremy Leach	Non-constant mean curvature trumpet solutions for the Einstein constraint equations		Submitted
Boaz Haberman	Unique determination of a magnetic Schrödinger operator with unbounded magnetic potential from boundary data		Posted
Nathan Totz	Global Well-Posedness of 2D non-focusing Schrodinger equations via Rigorous Modulation Approximation		Rough/Draft