Spring 2019

EMISSARY

Mathematical Sciences Research Institute

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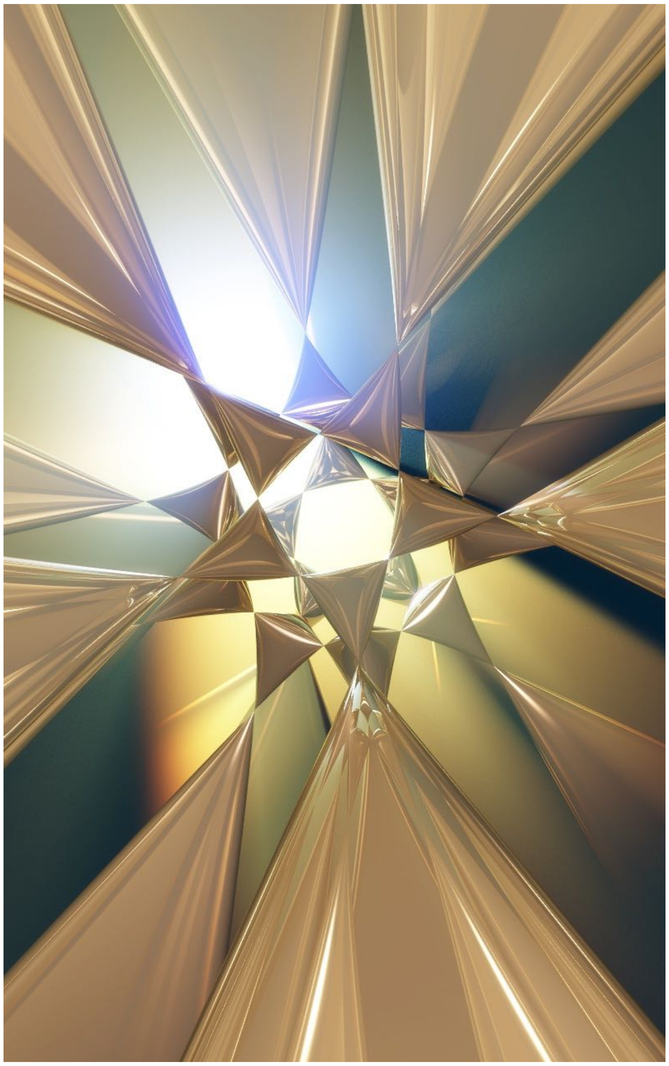


Birational Geometry and Moduli Spaces

János Kollár

The aim of the program in birational geometry and moduli spaces is to deepen our understanding of the following two problems.

> **Main Question 1.** Given an algebraic variety X, is there another variety X^m such that X and X^m are "similar" to each other, while the global geometry of X^m is the "simplest" possible?

> **Main Question 2.** Given a family of algebraic varieties $X \to S$, is there another family $X^c \to S^c$ such that $X \to S$ and $X^c \to S^c$ are "similar" to each other, while the global geometry of $X^c \to S^c$ is the "simplest" possible?

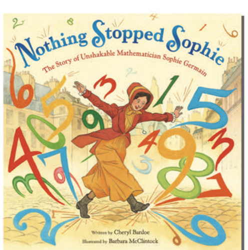
Much of algebraic geometry in the last two centuries has been devoted to answering these questions; frequently, a key point was to reach an agreement on what the words "variety," "similar," or "simplest" should mean.

For one-dimensional varieties, that is, for algebraic curves, the answer to the first question was given by Riemann (1857): the "simplest" algebraic curves are the smooth, compact ones. As for the "simplest" families, Riemann's suggestions for smooth families were made rigorous by Teichmüller (1944), but the complete answer was found only by Deligne–Mumford (1969).

(continued on page 10)

View of a Barth sextic obtained by applying a rotation involving the w (homogeneous) coordinate in order to make the 15 ordinary double points (the cone-like features) that are located on the plane at infinity visible. Image generated by Abdelaziz Naït-Merzouk.

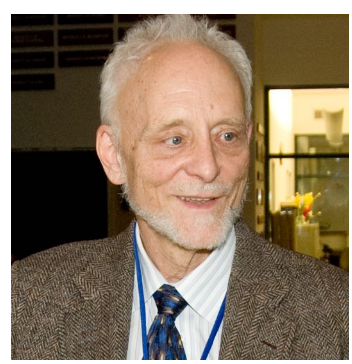
More Great Reading Inside



As always, there's lots more in the Spring Emissary — the perennial favorite Puzzles Column (page 13), the Derived Algebraic Geometry program (page 4), the story of a Numberphile-inspired tweet (page 7), a Q&A session with Cal Moore (page 16), and the very grateful 2018 Annual Report of donors (page 14). There are even a few recommendations for your summer reading list: the Mathical winners (one pictured here) on page 13; the newest Math Circles Library volume (page 7); and the Pacific Journal of Mathematics (page 5).

In Memoriam: Elwyn Berlekamp

As this issue was going to press we learned of the passing of our dear friend, Elwyn Berlekamp, on April 9, 2019. He was an integral part of MSRI and a great scientist, and will be deeply missed. We have created a page on our website to share memories of Elwyn — www.msri.org/elwyn.



The View from MSRI

Hélène Barcelo, Acting Director

It has now been more than half a year since I took on the role of Acting Director of MSRI, while David Eisenbud takes a semi-sabbatical that has turned out to be very busy indeed. Lately, we have been consumed by the preparation of our five-year NSF proposal, which David has tirelessly and expertly led. (We "pressed the submit button" last Friday [March 8], at 4:01 pm!)

MSRI's staff has been exemplary, putting up with our frequent demands of material "due yesterday" and with the hectic pace we set for ourselves; they have kept the institute running smoothly and reminded us that all runs well when we do not interfere! Assistant Director Michael Singer has been invaluable, keeping his cool and patient demeanor throughout this complex process. He has also been instrumental in shepherding our diversity initiatives and Human Resources Advisory Committee, among many other tasks. It has been a pleasure working with him, and I wish he could stay for many more years!

Meanwhile, the joint Connections for Women and Introductory workshops got our two Spring programs started. You can read more about what is happening in the Derived Algebraic Geometry program and the Birational Geometry and Moduli Spaces program further on in this issue. It is fair to say that everyone is busy collaborating and pushing the limits of our knowledge in these areas.

Survey of Postdocs Results

Since last fall, we have obtained the results of the Survey of our Postdoctoral Fellowship program, which covers the years 2009–16. This evaluation was performed by independent consultant, Amelia Taylor, with the help of MSRI's Grants and Data Specialist Alaina Drake-Moore, and Michael Singer. Postdoctoral training has been at the center of MSRI's activities from its very beginning in 1982.

Each year, MSRI invites about 35 postdoctoral fellows (who are at most five years beyond their Ph.D.) to participate in its scientific programs. These semester-long fellowships are highly prized: even though the applicants are self-selected by field, there are about ten times as many applicants as positions available, and nearly all of those to whom MSRI offers a fellowship actually attend the program. The postdoctoral program aims to:

- Introduce early-career mathematicians to the top researchers in their fields and allow them to integrate into that cohort of researchers;
- Allow them to expand and develop their research programs and encourage collaboration with their peers and more senior researchers;
- Provide carefully crafted mentorship by the leaders in these areas to develop fellows' professional and research skills, enabling them to become future leaders themselves.

We are happy with the results of the survey. Here are some highlights of what we found.

Professional status. Of the 139 former postdoctoral fellows (which corresponds to 68% of the surveyed population) who responded to the survey, 92% still engage in research activities in their current positions and 89% are currently employed in academia. Moreover, 65% of the respondents working in academia currently hold tenured or tenure-track positions. When asked to what extent their fellowship helped them secure a new or better position, 66% of respondents answered either "significantly" or "a great deal."

Research activity. The 139 respondents also reported an estimated 1,461 new professional contacts gained, 228 new co-authors gained, and 328 papers published as a result of their MSRI fellowship. Additionally, 78% of respondents said their experience as a postdoc at MSRI helped them either "significantly" or "a great deal" in developing new research directions; 91% said they still use knowledge and skills gained at MSRI in their current positions, at least occasionally.

Mentoring. Seventy percent of respondents rated the overall quality of mentoring they received at MSRI as either "excellent" or "very good." Respondents also provided many anecdotal comments that highlight the strengths of MSRI's mentoring program (for example, the fact that it is structured, yet flexible to suit the needs of the individual postdoc) as well as the immense benefit they received from more informal interactions with other senior researchers in residence at MSRI.

Equity. We were also pleased to see that MSRI is doing well with respect to equity among its postdoctoral fellows:

- Gender: Of MSRI's US-based fellows, 27% identified as women. In contrast, women held only about 20% of non-tenured postdoctoral positions at US institutions granting a Ph.D. in mathematics from 2009–16.
- Race and ethnicity: Among MSRI's fellows who were US citizens or permanent residents, 9% were Hispanic, 7.5% were Black, and 1.5% were Pacific Islanders. This compares to the population of new US/Puerto Rican Ph.D. recipients where only 4.3% were Hispanic, 3% were Black, and 0.5% were Pacific Islanders.

SWiM and ADJOINT

This summer will be the third summer that MSRI is running the Summer Research for Women in Mathematics (SWiM) program that brings groups of women to MSRI in order to push forward collaborations that they have previously started, perhaps at one of the

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conferences for women in particular areas, such as Women in Topology or Women in Combinatorics. (See the Fall 2018 Emissary for a detailed description of the goals and history of this program.)

Forty-six groups, comprising 153 women, applied for the 2019 program, up from 22 groups and 81 women who applied last year. Thanks to grants from the National Security Agency, the Lyda Hill Foundation, and Microsoft Research, we have been able to extend offers to 13 groups (46 women) for multiple two-week visits during the summer, more than doubling the number of last year's participants.

Inspired by the success of this program, MSRI is moving forward to meet the needs of other underrepresented groups. After discussions with Edray Goins of Pomona College and Robin Wilson of California State Polytechnic University, MSRI will run the African Diaspora Joint Mathematics Working Groups (ADJOINT) program during the summer of 2019.

Although modeled on the SWiM program, the ADJOINT program will also support teams that have a planned research program that they want to initiate, because, unlike the "Women in . . . " conferences, there are no such topic-focused programs for faculty in the African-American community. One goal of the ADJOINT program is to partially fulfill the role of these workshops, and also to facilitate having African-Americans participate more fully in the research programs of the various mathematics institutes.

The ultimate goal of both the SWiM and ADJOINT programs is to strengthen mathematics by facilitating the inclusion and visibility of talented researchers from these currently underrepresented groups.

Off-Site Summer Graduate Schools

You may know that MSRI has offered summer graduate schools for many years at our beautiful Berkeley facility. What you may not know is that there are many partnerships MSRI has formed around the country and the world to accommodate the high level of interest in our summer schools as well as to encourage the formation of strong connections between US students and students in other countries, paving the way for future collaborations.

Thanks to those partnerships, over the last ten years we have doubled the number of graduate schools that we offer each summer. For Summer 2019, we received almost 500 nominations from graduate chairs wanting to send their students to one of our ten summer schools; six of the schools will be held offsite in collaboration with the University of Notre Dame; Microsoft and the University of Washington, Seattle; the National Centre for Theoretical Sciences in Taipei, Taiwan; the Séminaire de Mathématiques Supérieures, Montréal, Canada; the Instituto de Matematicás, UNAM-Oaxaca, Oaxaca, Mexico; the Istituto Nazionale di Alta Matematica and Scuola Matematica Interuniversitaria, Cortona, Italy.

For 2020, 2021 and beyond, we are developing exciting summer schools in Canada, England, Greece, Switzerland, China, and Australia. You can read more about MSRI's Summer Graduate Schools, including how to apply, at msri.org/sgs.

With our semester-long programs, the onsite summer schools, MSRI-UP (MSRI's flagship REU program), and the SWiM and ADJOINT programs, I am happy to say that there is never a dull moment as MSRI is buzzing with activities all year round!

Members and Former Members Support MSRI

Let me conclude by expressing my heartfelt thanks to our large and generous pool of philanthropic donors, to the private and corporate foundations, to the NSF and the NSA, and to all of our past and current members who have given in different ways, including time and talent. Their gifts have enabled us to grow our offerings for scientific programming and individual support, thus providing a rich research environment to the national and international mathematical community.

CME Group–MSRI Prize

Albert S. Kyle

The 13th annual CME Group–MSRI Prize in Innovative Quantitative Applications was awarded to **Albert S. (Pete) Kyle**, Charles E. Smith Chair in Finance at the Robert H. Smith School of Business, University of Maryland, at The Chicago Cultural Center in Chicago in April 2019. A panel discussion on "Equity Market Design in the 21st Century" was held in conjunction with the award ceremony.

Dr. Kyle's research focuses on market microstructure, including topics such as high frequency trading, informed speculative trading, market manipulation, price volatility, the informational content of market prices, market liquidity, and contagion.

The annual CME Group–MSRI Prize is awarded to an individual or a group to recognize originality and innovation in the use of mathematical, statistical, or computational methods for the study of the behavior of markets, and more broadly of economics. You can read more about the prize at tinyurl.com/cme-msri.

MSRI-UP Students Earn Honors at MAA Poster Session

Just to share good news about the performance of our MSRI-UP 2018 students at the 2019 JMM in Baltimore this past January: Eight MSRI-UP students received honors at the MAA Student Poster Session, representing four different research projects completed in Summer 2018. Here are the honorees:

Cameron Hooper — Statistical Analysis and Geographical Clustering of Arrest Data for Los Angeles County

Esteban Escobar — Detection of Atrial Fibrillation in Electrocardiograms via Persistent Homology-based Features

Heyley Gatewood, **Samuel Hood**, and **Jonathan Scott** — TActIC: Tanh Activations in Image Classification

Skylyn Irby, **Nathalie Huerta**, and **Cristal Quiñones** — Statistical Analysis and Modeling of Exclusionary Discipline in K-12 California Public Schools

Congratulations to our MSRI-UP representatives and to all students who presented at the conference!

Derived Algebraic Geometry

Akhil Mathew

One goal of algebraic geometry is to study algebraic varieties, that is, the solution sets of polynomial equations in many variables. This can take many forms:

1. Given a variety over the complex numbers $\mathbb{C}$, one can study its topology (and invariants such as cohomology).
2. Given a variety with coefficients in the integers $\mathbb{Z}$ or a finite field $\mathbb{F}_q$, one can study its rational points (e.g., counting them).
3. Given a variety (over any base), one can study its deformations and moduli: how it varies in families.

Commutative rings are the building blocks for algebraic varieties and more generally schemes. The subject of derived algebraic geometry involves a generalization of the notion of a variety (or scheme) to a *derived scheme.* While ordinary schemes are modeled on the spectra of commutative rings, derived schemes are modeled on derived versions of commutative rings, for instance the theory of simplicial commutative rings initiated by Quillen, or the more topological theory of $\mathbb{E}_\infty$-ring spectra in stable homotopy theory.

Intuitively, one can think of the relationship between derived schemes and ordinary schemes as analogous to the relationship between ordinary schemes and reduced schemes (that is, corresponding to rings without nilpotents). Derived algebraic geometry thus yields a generalization of classical algebraic geometry. In many cases, derived algebraic geometry has been successful in shedding light on classical questions such as the above, by introducing new tools and techniques.

This semester's Derived Algebraic Geometry program features a mix of researchers applying derived techniques in a variety of settings, ranging from chromatic homotopy theory to representation theory to p-adic geometry. Here we touch on only a few such directions.

Why Derived Algebraic Geometry?

Even for problems involving classical commutative rings, techniques of derived algebraic geometry can often provide additional information. An instance of this is the work starting with André–Quillen in the 1970s on the cotangent complex (a derived functor version of differential forms, and an important precursor to modern derived algebraic geometry) culminating in the characterization by Avramov of local complete intersections via André–Quillen homology. These ideas rely on the cotangent complex, but not perhaps on the full strength of derived algebraic geometry.

Natural operations on classical commutative rings or ordinary schemes can give also rise to derived rings. A celebrated example of this is Serre's intersection formula for local intersection multiplicities, where the higher Tor functors (the homology groups of a simplicial commutative ring) arise in formulating Bezout's theorem for non-transverse intersections. The general principle is that any non-transverse intersection of subvarieties has a natural derived structure, which remembers additional information about the intersection (for example, an intersection of a smooth subvariety with itself should remember the normal bundle).

In moduli theory, methods of derived algebraic geometry often yield both simpler and richer statements: derived moduli problems are easier to classify and work with, and contain additional information. A moduli problem considers families of some type of algebro-geometric object (for example, varieties, vector bundles, maps, ...) and is often classically parametrized by a moduli variety or stack. Studying such families over derived rings provides additional structure to the moduli problem.

A consequence is that one can sometimes obtain general classifications. In characteristic zero, the infinitesimal neighborhood of a *derived* moduli space can be completely determined via a differential graded Lie algebra by the work of Lurie and Pridham, formalizing a principle due to Deligne, Drinfeld, Feigin, and others. The derived enhancement of a classical moduli problem also contains a key piece of additional data: the derived structure sheaf. This can be used sometimes to extract concrete numerical invariants, for example, in the theory of virtual fundamental classes as in the work of Behrend–Fantechi and Li–Tian.

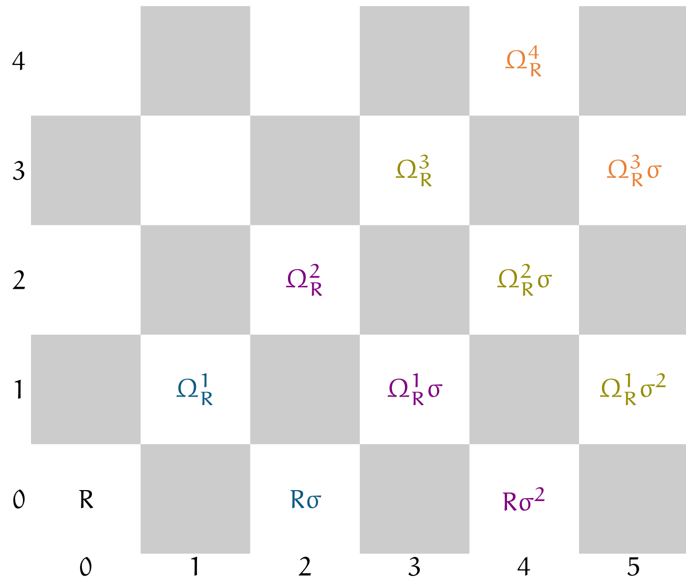
The subject of derived algebraic geometry necessarily involves a fusion of methods and ideas both from homotopy theory and algebraic geometry. For example, the basic objects of derived algebraic geometry (derived rings and schemes) are most naturally organized into a higher category, rather than simply an ordinary category. The language of ∞-categories (or quasicategories) developed by Joyal and Lurie provides an efficient framework for manipulating such objects and is essential to many recent developments in the subject.

Topological Hochschild Homology

Topological Hochschild homology (THH) is an analog, introduced by Bökstedt, of classical Hochschild homology in the setting of spectrally derived algebraic geometry. For an associative algebra A over a field k, Hochschild homology is the homology of an explicit complex involving the iterated tensor powers of A. One remarkable feature of Hochschild homology is the Hochschild–Kostant–Rosenberg theorem, which states that when one inputs a smooth commutative algebra over k, then the Hochschild homology groups recover the modules of differential forms of A/k. Using the Connes–Tsygan differential on Hochschild homology, one can also recover de Rham cohomology by this approach.

The idea of topological Hochschild homology is that one should replace the base field k by the "smallest" possible base, in the hope of obtaining a richer and deeper invariant. In ordinary (underived) algebra, the initial ring is of course the integers $\mathbb{Z}$, but it is observed that Hochschild homology relative to $\mathbb{Z}$ does not generally produce nice invariants: for instance, the Hochschild homology of $\mathbb{F}_p$ over $\mathbb{Z}$ is a divided power algebra. In higher algebra, the initial object is the sphere spectrum $\mathbb{S}$, and topological Hochschild homology can be defined as Hochschild homology relative to the base $\mathbb{S}$.

It is a remarkable fact that the construction of topological Hochschild homology, natural from the point of view of homotopy theory, produces deep and intricate arithmetic structures some of which are only now being explored and understood. More generally, the idea is that THH (and its variants) of a p-adic ring should be related to p-adic cohomology theories. The starting point is Bökstedt's theorem that $\mathrm{THH}_*(\mathbb{F}_p)$ is a polynomial ring $\mathbb{F}_p[\sigma]$ on a generator σ of degree two, which already shows that it produces nicer answers than Hochschild homology over $\mathbb{Z}$. The work of Hesselholt showed that the de Rham–Witt complex of Bloch–Deligne–Illusie arises as the homotopy groups of constructions built from THH for smooth $\mathbb{F}_p$-algebras.



The topological Hochschild homology (THH) of a smooth $\mathbb{F}_p$-algebra, and the Bhatt–Morrow–Scholze filtration.

An exciting recent development surrounding THH has been the work of Bhatt–Morrow–Scholze on integral p-adic Hodge theory, which pushes these results into mixed characteristic and sharpens them. For smooth schemes over $\mathcal{O}_C$ (the ring of integers in the field $C = \mathbb{C}_p$), they show that one can naturally construct from THH and its variants a filtration whose associated graded quotients give rise to p-adic cohomology theories (called A_{inf}-cohomology) which interpolate between the de Rham cohomology and the étale cohomology of the generic fiber. The construction of the Bhatt–Morrow–Scholze filtration relies essentially on various tools in derived algebraic geometry, notably the cotangent complex, and heavily uses Scholze's perfectoid rings. It is expected that this filtration is a deep and fundamental construction, which should be connected to the motivic filtration relating algebraic K-theory to motivic cohomology. A visualization of the filtration on topological Hochschild homology in characteristic p is shown in the figure above.

Recently, Bhatt and Scholze have given a new site-theoretic construction and generalization of A_{inf}-cohomology called prismatic cohomology, which avoids the use of THH by using instead the theory of δ-rings, or rings with a "p-derivation." The theory of δ-rings and prismatic cohomology, while avoiding the machinery of THH, plays also an important role in $K(1)$-local homotopy theory via power operations on $\mathbb{E}_\infty$-ring spectra, and one expects it to play a growing role in derived algebraic geometry.

Elliptic Cohomology: Derived Algebraic Geometry in Algebraic Topology

Ideas from derived algebraic geometry have also powerfully influenced modern algebraic topology, via the theory of topological modular forms. A basic tool in algebraic topology and stable homotopy theory is the use of *cohomology theories*. The first example of a cohomology theory is given by ordinary singular cohomology. Beyond that, one next typically encounters *complex* K-*theory*; this is a cohomology theory $\{K^*(\cdot)\}$ such that, for a compact CW complex X,

$$K^i(X) = \begin{matrix}\text{The Grothendieck group}\\ \text{of vector bundles on}\end{matrix} \begin{cases} X & \text{if } i \text{ is even} \\ \Sigma X & \text{if } i \text{ is odd} \end{cases}.$$

Constructing exotic cohomology theories has long been of interest to topologists seeking to solve questions involving the stable homotopy groups of spheres, such as Adams–Atiyah's "postcard" solution of the Hopf invariant one problem via K-theory. Remarkably, the technology of derived algebraic geometry provides powerful tools for building newer and more powerful cohomology theories.

Much research in stable homotopy theory over the past couple of decades has focused on the object of topological modular forms (TMF), constructed originally by Goerss, Hopkins, and Miller, and later by Lurie, as the global sections of a sheaf of $\mathbb{E}_\infty$-rings on the moduli stack of elliptic curves. This results in an extremely intricate elliptic spectral sequence computing the homotopy groups of TMF given by

$$H^i(M_{ell}, \omega^{\otimes j}) \implies \pi_{2j-i}(\mathrm{TMF}).$$

A key theme in both constructions is that it is not enough to construct TMF simply as a cohomology theory: rather, one must construct it as an $\mathbb{E}_\infty$-ring, that is, as an analog of a commutative ring in the context of spectrally derived algebraic geometry.

The work of Lurie takes this picture further using the derived version of the Artin representability theorem. In particular, the derived version of the moduli stack of elliptic curves that yields TMF is constructed as the solution to a moduli problem defined on $\mathbb{E}_\infty$-rings, and thus endows TMF with a universal property. The perspective of derived algebraic geometry has led to additional advances in the understanding of TMF, such as the work by Hill–Lawson on topological modular forms with level structure constructed using the log-étale site and the work of Behrens–Lawson on topological automorphic forms, and continues to inform further work both theoretical and computational in homotopy theory.

Focus on the Scientist: Lars Hesselholt

Lars Hesselholt is a professor at Nagoya University and at the University of Copenhagen, and previously spent 15 years at MIT. He received his Ph.D. from Aarhus University in 1994. His work has been recognized by numerous honors, including the Clay Senior Scholarship at MSRI in Spring 2014 and the Niels Bohr Professorship at the University of Copenhagen.

Lars Hesselholt

Lars is an algebraic topologist, and is *the* leader in the field of algebraic K-theory: his work has changed the face of the subject and inspired new breakthroughs (not just in K-theory, but also further areas such as p-adic Hodge theory).

Algebraic K-theory is a cohomology theory in algebraic geometry invented by Quillen (motivated by the work of Atiyah–Hirzebruch in topology, which was itself inspired by Grothendieck's notion of K_0 in algebraic geometry). Unlike other cohomology theories (such as étale cohomology), this theory is notoriously difficult to compute. Nevertheless, computations of K-theory are of fundamental importance: K-theory lies at the heart of some of the deepest conjectures in arithmetic geometry (such as Beilinson's conjectures, linking K-groups to L-functions).

Lars is a premier exponent of using "trace methods" in computing K-theory, including using the Dundas–Goodwillie–McCarthy theorem to reduce K-theory calculations to those in topological cyclic homology. Lars has wielded this approach to great effect in computing K-theory. As just one example, we mention his work with Madsen, using this approach to verify the Lichtenbaum–Quillen conjecture for the K-groups of complete discretely valued fields of mixed characteristic.

In recent years, Lars has also turned his attention more directly towards the interactions between K-theory and arithmetic geometry. For example, in 2006, Lars showed that the topological cyclic homology of the ring of integers of an algebraically closed p-adic field is closely related to a fundamental construction of Fontaine in p-adic Hodge theory. This calculation provided the first example where objects of interest in number theory were found to be closely related to the sphere spectrum — it was a stunning vindication of Waldhausen's philosophy that working over the sphere spectrum (that is, in "higher algebra") instead of the integers gives better-behaved answers.

Besides being a stellar researcher, Lars is a fantastic lecturer. We simply give an anecdote (from Haynes Miller) to illustrate his popularity with students: "Once he walked into class in his khaki slacks, balancing a cup of coffee on his notes, and saw the entire class standing at their seats, in slacks, balancing cups of coffee on their notes."

— Bhargav Bhatt

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are **March 15**, **October 15**, and **December 15**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by **March 15**, **October 15**, or **December 15** for review at the upcoming SAC meeting. See tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several two-week long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by **March 15**, **October 15** or **December 15** for review at the upcoming SAC meeting. See tinyurl.com/msri-sgs.

Call for Membership

MSRI invites membership applications for the 2020–21 academic year in these positions:

Research Professors	by October 1, 2019
Research Members	by December 1, 2019
Postdoctoral Fellows	by December 1, 2019

In the academic year 2020–21, the research programs are:

Random and Arithmetic Structures in Topology
Aug 17 – Dec 18, 2020
Organized by Nicolas Bergeron, Jeffrey Brock, Alexander Furman, Tsachik Gelander, Ursula Hamenstädt, Fanny Kassel, Alan Reid

Decidability, Definability, and Computability in Number Theory
Aug 17 – Dec 18, 2020
Organized by Valentina Harizanov, Maryanthe Malliaris, Barry Mazur, Russell Miller, Jonathan Pila, Thomas Scanlon, Alexandra Shlapentokh, Carlos Videla

Mathematical Problems in Fluid Dynamics
Jan 19 – May 28, 2021
Organized by Thomas Alazard, Hajer Bahouri, Mihaela Ifrim, Igor Kukavica, David Lannes, Daniel Tataru

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at mathjobs.org after August 1, 2019. For more information about any of the programs, please see msri.org/programs.

Public Understanding of Mathematics — Highlights

Congressional Briefings

MSRI and the American Mathematical Society (AMS) host two Congressional briefings on mathematical topics each year in Washington, DC, to inform members of Congress and Congressional staff about new developments made possible through federal support of basic science research. On December 4, 2018, **Rodolfo H. Torres** (pictured), University Distinguished Professor of Mathematics at the University of Kansas, spoke to Congressional staff and the public about his research.

In his talk — "From the Color of Birds to Nanomaterials and New Technology" — Torres explained how investigating the beautiful coloration of bird feathers with mathematical tools, including Fourier analysis, has led scientists to new technologies in the fabrication of materials of highly saturated colors, adaptive camouflage properties, and efficient photovoltaic attributes. Learn more and view upcoming events: msri.org/congress.

2019 National Math Festival

The 2019 National Math Festival will take place on Saturday, May 4, in Washington, DC (nationalmathfestival.org).

Math Organizations Join the 2019 Festival. A new addition to the National Math Festival, the Make or Take Spiral features hands-on activities and take-home resources for festival visitors of all ages. Participating organizations include the Art of Problem Solving, Association for Women in Mathematics (AWM), several DC-area Math Circles, members of the DREME (Development and Research in Early Math Education) Network, the Erikson Institute Early Math Collaborative, Math Monday, Mathematical Association of America (MAA), National Association of Mathematicians (NAM) and the Benjamin Banneker Association, National Council of Teachers of Mathematics (NCTM), NOVA, the National Science Foundation (NSF), ThinkFun Games, and WGBH.

Ithaca College Department of Mathematics returns with the beloved Geometric Balloon Bending stations, and many presenters, including the Julia Robinson Mathematics Festival, Natural Math, the Young People's Project, will also offer materials in addition to their Festival programming. The 2019 National Math Festival is organized by MSRI in cooperation with the Institute for Advanced Study (IAS) and the National Museum of Mathematics (MoMath).

After the Festival. Immediately following the 2019 Festival in Washington, DC, the National Museum of the American Indian will host a public screening of MSRI's 2016 documentary ***Navajo Math Circles***. The screening includes special guests director George Csicsery, Tatiana Shubin (San Jose State University), and Natanii Yazzie, one of the Navajo students featured in the film.

Around the US. MSRI is partnering with the Association of Science-Technology Centers (ASTC) and Zometool to offer **Zometool Bubble Stations** at over 80 museums in 42 US states and territories on the 2019 Festival Day, May 4. Museum visitors will explore how minimal surfaces make possible not just the round soap bubbles we all know from our childhoods, but — with differently shaped Zometool "bubble frames" — geometric soap bubbles of many surprising forms.

Numberphile's Reach

Michael Foskett, a Ph.D. student at the University of Surrey, was part of last fall's Hamiltonian Systems program. While here, he discovered the plaque that celebrates Numberphile's one-millionth subscriber on YouTube and tweeted this picture.

"It was amazing to see it, because Numberphile is one of the reasons I decided to study maths at university. I was always fascinated by nature and science, and Numberphile was relatable and really interesting and presented these ideas in short, super interesting clips — and not just about one area of maths. It made me see maths from a broader perspective and get a deeper understanding of everything, as I realized that maths explains the world."

Events in the Bay Area and Beyond

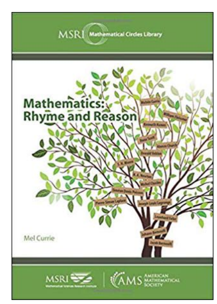
Here are some highlights of other outreach events hosted by MSRI so far in 2018–19:

Free **Harmonic Series concerts** featuring pianist Nicholas Tjahjono and the Jeff Denson Three jazz ensemble.

Art at MSRI: Two **art receptions** were held, featuring photographs from the "Airplane as Art" collection by Robert "Bob" Seidemann, and a joint mixed media exhibit with works by Virginia Davis and MSRI's own Claude Ibrahimoff.

MSRI and the Fleet Science Center hosted a **public lecture in San Diego** by Tadashi Tokieda (Stanford University) on "The World from a Piece of Paper" in October 2018, exploring a variety of phenomena, from magic tricks and geometry to nonlinear elasticity and the art of origami.

Math Circles Library



The latest volume in the Mathematical Circles Library series has just been released. MSRI and the AMS publish the series as a service to young people, their parents and teachers, and the mathematics profession. *Mathematics: Rhyme and Reason*, by Mel Currie, explores the aesthetic value of mathematics and the culture of the mathematics community. Part personal memoir, part appreciation of the poetry and humanity inherent in mathematics, this entertaining collection of stories, theorems, and reflections introduces budding mathematicians of all ages to mathematical ways of thinking. Explore the entire collection of MSRI Mathematical Circles Library titles at bookstore.ams.org/MCL.

Named Postdocs — Spring 2019

Berlekamp

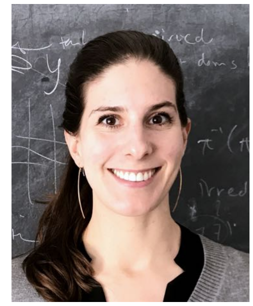
Piotr Achinger, a member of the Derived Algebraic Geometry program, is the Spring 2019 Berlekamp Postdoctoral Fellow. Piotr completed his undergraduate studies in mathematics and computer science at the University of Warsaw, in his home city of Warsaw, Poland. During his Ph.D. studies at the University of California, Berkeley, under the supervision of Arthur Ogus, he was supported by a Fulbright Science and Technology Fellowship. He spent the first two years after his Ph.D. as a postdoc funded by the European Post-Doctoral Institute (EPDI) between the Banach Center in Warsaw and Institut des Hautes Études Scientifiques (IHES) in Bures-sur-Yvette, France. He currently works as an Assistant Professor at the Institute of Mathematics of the Polish Academy of Sciences, where he is the PI of a 5-year European Research Council Starting Grant. Piotr's research focuses on the topology of algebraic varieties, mainly over fields of positive characteristic. *The Berlekamp Postdoctoral Fellowship was established in 2014 by a group of Elwyn Berlekamp's friends, colleagues, and former students whose lives he touched in many ways. He is well known for his algorithms in coding theory and made important contributions to game theory. He is also known for his love of mathematical puzzles.*

Stephen Della Pietra

Lukas Brantner is the Stephen Della Pietra Fellow in this semester's Derived Algebraic Geometry program and a Junior Research Fellow at Merton College, Oxford. Lukas was born and raised in Freiburg in Germany. As an undergraduate student at Cambridge, he worked on geometric group theory with Danny Calegari. He also studied Simpson's nonabelian Hodge theory under the guidance of Ian Grojnowski, which sparked his interest in derived algebraic geometry. This led Lukas to pursue his Ph.D. studies at Harvard under the direction of Jacob Lurie. In his dissertation, he examined the Lubin–Tate theory of spectral Lie algebras. More recently, in joint work with Akhil Mathew, Lukas has extended the formal deformation theory of Lurie–Pridham (inspired by work of Drinfeld, Hinich, and others) from characteristic zero to characteristic p. Aside from mathematics, Lukas enjoys sailing and playing the violin. *The Stephen Della Pietra fellowship was established in 2017 by the Della Pietra Family Foundation. Stephen received his Ph.D. in mathematical physics from Harvard University. He is a partner at Renaissance Technologies, a board member of the Simons Center for Geometry and Physics, and treasurer of the National Museum of Mathematics in New York.*

Gamelin

Kristin DeVleming is the Gamelin Postdoc in the Birational Geometry and Moduli Spaces program this semester. Kristin is also an NSF Research Training Group postdoc at the University of California, San Diego. Kristin is a very active and prolific mathematician with many different interests. She received her Ph.D. from the University of Washington in Seattle in 2018. Her thesis is on compact moduli of surfaces in 3-dimensional projective space. Kristin has also recently worked on hyperbolicity and positivity of log cotangent sheaves, in collaboration with Kenneth Ascher and Amos Turchet; interpolation between GIT and stable compactifications of moduli spaces plane curves using K-stability with Kenneth Ascher and Yuchen Liu; and two projects with David Stapleton on analogues of MRC fibrations and the generalized Bloch conjecture. Kristin is also very active in outreach activities, including work with Math Circles in Seattle (and in Berkeley this semester!), and with the Math Summer Camp for high schoolers in Seattle. *The Gamelin postdoctoral fellowship was created in 2014 by Dr. Ted Gamelin, Emeritus Professor of the UCLA Department of Mathematics. The Gamelin Fellowship emphasizes the important role that research mathematicians play in the discourse of K-12 education.*

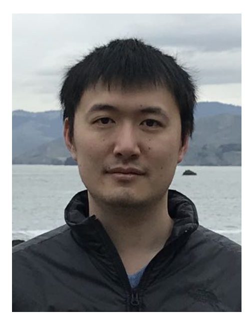


Viterbi

Tina Kanstrup is the Viterbi Postdoctoral Fellow in the Derived Algebraic Geometry program. Tina earned her Ph.D. in 2015 under the supervision of Sergey Arkhipov. Her research is in geometric representation theory, a field whose general goal is to express classical objects in representation theory in terms of constructions from (derived) algebraic geometry. One important example is the derived category of equivariant coherent sheaves on the Steinberg variety, which categorifies the affine Hecke algebra. This category receives a map from the affine braid group, a fact that has profound applications also outside of geometric representation theory. For example, one of Tina's current projects is to apply this affine braid group construction to a conjecture (due to Gorsky, Negut, and Rasmussen) in knot theory that expresses the Khovanov–Rozansky triply-graded link homology in terms of coherent sheaves on Hilbert schemes. *The Viterbi postdoctoral fellowship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.*

Vincent Della Pietra

Yuchen Liu is the Vincent Della Pietra Postdoc in this semester's Birational Geometry and Moduli Spaces program. He got his bachelor's and master's degrees at Peking University. Then he received his Ph.D. in 2017 from Princeton University, under the supervision of János Kollár. He now holds the position of a Gibbs Assistant Professor at Yale University. Yuchen mainly studied birational geometry. He has made important contributions to both local and global K-stability problems by applying the minimal model program. For example, he (joint with Blum) proved the lower semi-continuity for the normalized



volume as well as the delta invariant of Fano varieties. He also (joint with Xu) solved the existence of Kähler–Einstein metric for smooth cubic threefolds. *The Vincent Della Pietra fellowship was established in 2017 by the Della Pietra Foundation. Vincent received his Ph.D. in mathematical physics from Harvard University. He is a partner at Renaissance Technologies, co-founder of the Della Pietra Lecture Series at Stony Brook University, and a board member of PIVOT.*

Uhlenbeck

Thomas McConville is the Uhlenbeck Postdoctoral Fellow in the complementary program during the academic year 2018–19. After receiving his Ph.D. in 2015 from the University of Minnesota under the supervision of Pavlo Pylyavskyy, he spent three years as an Instructor of Applied Mathematics at MIT. In the fall of 2017 he was also a research member in the MSRI program on Geometric and Topological Combinatorics. Thomas' research in combinatorics builds on algebra as well as geometric and topological methods. In particular, he combines combinatorics and Coxeter group theory to gain a deep understanding of fundamental geometric objects such as associahedra. In his work the rich interplay among polytope theory, triangulations and combinatorics is employed to unravel connections that reach out to representation theory and topology. In their analysis of chip firing games, he and his coauthors employ the same paradigm in the study of discrete dynamical systems. *The Uhlenbeck fellowship was established by an anonymous donor in honor of Karen Uhlenbeck, a distinguished mathematician and former MSRI trustee. She is a member of the National Academy of Sciences and a recipient of the 2019 Abel Prize, the AMS Leroy P. Steele Prize, and a MacArthur "Genius" Fellowship.*

McDuff

Joseph Allen Waldron is the McDuff Postdoctoral Fellow for the Birational Geometry and Moduli Spaces program. He works on the birational geometry of varieties over a field of positive characteristic. Joe was an undergraduate student at Queens' College, Cambridge. He continued his studies there, obtaining a Ph.D. working under the direction of Caucher Birkar. After spending a year at the École polytechnique fédérale de Lausanne (EPFL) working with Zsolt Patakfalvi, he is now a postdoc at Princeton University working with János Kollár. He has an interest in photography, especially taking pictures of wildlife and birds. His early work was devoted to extending the minimal model program to characteristic p for threefolds. He is currently thinking about generic smoothness for Mori fibre spaces and extending the minimal model program to other settings, such as mixed characteristic. *The McDuff fellowship was established by an anonymous donor in honor of Dusa McDuff. She is an internationally renowned mathematician, a member of the National Academy of Sciences, and a recipient of the AMS Leroy P. Steele Prize (2017). She is also currently a trustee of MSRI.*

Named Positions

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Chern, Eisenbud, and Simons Professors

Derived Algebraic Geometry
Dmytro Arinkin, University of Wisconsin
Bhargav Bhatt, University of Michigan
Hélène Esnault, Freie Universität Berlin
Laurent Fargues, CNRS / Institut de Mathématiques de Jussieu
Lars Hesselholt, Nagoya University and University of Copenhagen

Birational Geometry and Moduli Spaces
Valery Alexeev, University of Georgia
Lucia Caporaso, Roma Tre University
Hélène Esnault, Freie Universität Berlin
Christopher Hacon, University of Utah
James M^{c}Kernan, University of California, San Diego
David Morrison, University of California, Santa Barbara
Chenyang Xu, MIT

Named Postdoctoral Fellows

Derived Algebraic Geometry
Berlekamp: Piotr Achinger, Polish Academy of Sciences
S. Della Pietra: Lukas Brantner, Merton College, Oxford University
Viterbi: Tina Kanstrup, Aarhus University

Birational Geometry and Moduli Spaces
Gamelin: Kristin DeVleming, University of California, San Diego
V. Della Pietra: Yuchen Liu, Yale University
McDuff: Joseph Waldron, Princeton University

Complementary Program
Uhlenbeck: Thomas McConville, MIT

Clay Senior Scholars

The Clay Mathematics Institute (www.claymath.org) has announced the 2018–19 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in a topical program at an institute or university away from their home institution. Here are the Clay Senior Scholars who will work at MSRI in 2019–20.

Holomorphic Differentials in Mathematics and Physics (Fall 2019)
Anna Wienhard, Universität Heidelberg

Microlocal Analysis (Fall 2019)
Gunther Uhlmann, University of Washington

Quantum Symmetries (Spring 2020)
Dan Freed, University of Texas at Austin

Higher Categories and Categorification (Spring 2020)
Peter Teichner, Max-Planck-Institut für Mathematik

Birational Geometry and Moduli Spaces

(continued from page 1)

In higher dimensions every variety is "similar" to a smooth one (Hironaka, 1966). While the local structure of a smooth variety is as simple as possible, the global structure is usually very complicated. One of the hardest aspects of the theory was to understand the optimal trade off between "simplest" global structure and "simplest" local structure. A series of conjectures — called the *minimal model program* or *Mori's program* — was developed by Mori and Reid around 1980.

A salient feature of algebraic geometry is that by continuously varying the coefficients of the defining polynomials we get continuously varying families of algebraic varieties. We can thus study how to transform a family $\{X_s : s \in S\}$ of varieties into its "simplest" form.

For a family of varieties $X \to S$, it is tempting to take the fibers $\{X_s : s \in S\}$ and their "simplest" models $\{X_s^m : s \in S\}$ obtained previously. This works over a dense open subset of S, but breaks down exactly at the most interesting cases where the structure of the fibers changes. An approach was proposed by Mumford (1965), but it seems that the method outlined by Kollár–Shepherd-Barron (1988) gives the right answer.

Thus, for both main questions, we believe that we have the right conjectures: these have been proved in many cases, but we are still working on complete solutions. In the rest of these notes I aim to explain what the main objects and conjectures are, and how we try to solve them.

What are Algebraic Varieties?

The basic objects of algebraic geometry are polynomials and their zero sets. An *affine algebraic set* in $\mathbb{C}^N$ is the common zero-set of some polynomials

$$\begin{aligned} X^{\text{aff}} &= X^{\text{aff}}(f_1,\dots,f_r) \\ &= \big\{(x_1,\dots,x_N) : f_1(\mathbf{x}) = \cdots = f_r(\mathbf{x}) = 0\big\} \subset \mathbb{C}^N. \end{aligned}$$

It is especially easy to visualize *hypersurfaces* $X(f) \subset \mathbb{C}^N$ defined by one equation. An affine algebraic set X^{aff} is almost never compact, so we usually work with the closure of X^{aff} in the complex projective space $X := X^{\text{proj}} \subset \mathbb{CP}^N$. Thus we get *projective algebraic sets.* X is a *projective variety* if it can not be written as the union of two projective algebraic sets in a nontrivial way. For example, a hypersurface $X(f)$ is a variety iff f is a power of an irreducible polynomial. We count complex dimensions: thus $\dim \mathbb{C}^N = N$, and $\dim X$ is one-half of the topological dimension of X. In low dimensions we talk about *curves, surfaces, 3-folds.* Somewhat confusingly, an algebraic curve is the same as a Riemann surface.

On $\mathbb{C}^N$ our basic functions are polynomials. On $\mathbb{CP}^N$ we have homogeneous coordinates $(x_0{:}\cdots{:}x_N)$ that are defined only up to multiplication by a scalar. Thus one cannot evaluate a polynomial $p(x_0,\dots,x_N)$ at a point of $\mathbb{CP}^N$. However, if p_1, p_2 are homogeneous of the same degree d, then $f(\mathbf{x}) := p_1(\mathbf{x})/p_2(\mathbf{x})$ is well-defined (except where p_2 vanishes). These are the *rational functions* on $\mathbb{CP}^N$. By restriction, we get rational functions on any projective variety $X \subset \mathbb{CP}^N$.

Birational Maps and Birational Equivalence

The oldest example of a birational map is the stereographic projection invented by Hipparchus (~190–120 BC) to create star charts. We project the unit sphere $\mathbb{S}^n := \big(x_1^2 + \cdots + x_{n+1}^2 = 1\big) \subset \mathbb{R}^{n+1}$ from the south pole $\mathbf{p}_0 := (0,\dots,0,-1)$ to the plane $x_{n+1} = 0$, where we use $y_1,\dots,y_n$ as coordinates. Geometrically, pick any point $\mathbf{p} \in \mathbb{S}^n$ (other than the south pole) and let $\pi(\mathbf{p})$ denote the intersection point of the line $\langle \mathbf{p}_0, \mathbf{p}\rangle$ with the hyperplane $(x_{n+1} = 0)$. Algebraically, we get that

$$\pi(\mathbf{x}) = \big(\tfrac{x_1}{1+x_{n+1}},\dots,\tfrac{x_n}{1+x_{n+1}}\big), \text{ and}$$

$$\pi^{-1}(\mathbf{y}) = \big(\tfrac{2y_1}{1+\Sigma},\dots,\tfrac{2y_n}{1+\Sigma},\tfrac{1-\Sigma}{1+\Sigma}\big),$$

where $\Sigma = y_1^2 + \cdots + y_n^2$. Generalizing this example, we say that two algebraic varieties $X_1 \subset \mathbb{P}^{N_1}$ and $X_2 \subset \mathbb{P}^{N_2}$ are *birational* to each other if there are maps $\pi : X_1 \dashrightarrow X_2$ and $\pi^{-1} : X_2 \dashrightarrow X_1$ such that the coordinate functions of π and π^{-1} are rational. Composing with π and π^{-1} shows that we can identify rational functions on X_1 with rational functions on X_2. For our purposes, "similar" = birational.

The Canonical Class

In order to define what a "simple" global structure is, we need the notion of the *canonical class.* (There is a persistent sign problem in this area since K_X is the negative of the first Chern class of X.)

A measure on $\mathbb{R}^n$ is an n-form $s(x_1,\dots,x_n)\cdot dx_1 \wedge \cdots \wedge dx_n$. Similarly, on a smooth complex variety X of dimension n, a *complex volume form* is an n-form ω that locally can be written as $h(z_1,\dots,z_n)\cdot dz_1 \wedge \cdots \wedge dz_n$. Thus $\omega \wedge \bar\omega$ is a real volume form (up to a power of $\sqrt{-1}$). Complex volume forms can be viewed as sections of a line bundle, called the *canonical bundle,* usually denoted by K_X.

There are two ways to compute the *degree* of K_X on an algebraic curve $C \subset X$.

Working algebraically, one can choose h and the z_i to be rational functions and obtain an algebraic volume form σ that is defined and nonzero at all but finitely many points of C. We define the *degree* or *intersection number* of K_X on C as

$$(K_X \cdot C) := \#(\text{zeros of } \sigma \text{ on } C) - \#(\text{poles of } \sigma \text{ on } C),$$

both counted with multiplicities. Differential geometers would choose a C^∞ volume form $\omega = h\cdot dz_1 \wedge \cdots \wedge dz_n$ and define its *first Chern form* as

$$c_1(\omega) := \frac{\sqrt{-1}}{\pi}\sum_{ij}\frac{\partial^2 \log|h(\mathbf{z})|}{\partial z_i \partial \bar z_j} dz_i \wedge d\bar z_j.$$

The bridge between the two notions is the Gauss–Bonnet theorem which says that for every algebraic curve $C \subset X$,

$$(K_X \cdot C) = \int_C c_1(\omega).$$

On a typical variety, K_X is positive on some curves and negative on others. The "simplest" varieties are those where K_X has a sign.

The Three Basic Types of Varieties

1. K**-positive or general type.** These are the varieties where $(K_X \cdot C)$ is positive for every curve $C \subset X$. This is the largest class of the three.

2. **Flat or Calabi–Yau.** Here $(K_X \cdot C)$ is zero for every curve $C \subset X$. They play an especially important role in string theory and mirror symmetry.

3. K**-negative or Fano.** Here $(K_X \cdot C)$ is negative for every curve. There are few of these but they occur frequently in applications.

We also need two more mixed types:

4. K**-semipositive or Kodaira–Iitaka type.** Here $(K_X \cdot C) \geqslant 0$ for every curve $C \subset X$ and there is a unique morphism $I_X : X \to I(X)$ — called the *Iitaka fibration* — such that $(K_X \cdot C) = 0$ iff C is contained in a fiber of I_X.

5. **Mori fiber space.** There is a morphism $m_X : X \to M(X)$ such that $(K_X \cdot C) < 0$ if C is contained in a fiber of m_X.

We can now state a more precise version of the first main question:

Conjecture. (Minimal model conjecture.) Every algebraic variety X is birational to a variety X^m that is one of the types (1–5).

A strong caveat is that one has to allow certain singularities on X^m. Depending on the cases, these are the *canonical* and *terminal* singularities.

The cases $\dim X = 2, 3$ are mostly due to Castelnuovo and Enriques (1898–1947), and to Mori (1982–88). In higher dimensions, roughly speaking, the answer is known for those varieties for which X^m is expected to be of type (1), (3) or (5); this is mostly the work of Hacon–McKernan (2007–10).

In the K-positive case X^m is unique, but not in the other cases; uniqueness in cases (3) and (5) is an especially difficult topic of intense research.

Moduli Spaces of Varieties

We start the discussion of families of varieties with the historically first example, though in retrospect its special features may have hindered progress for a while.

Example. (Moduli of elliptic curves). An *elliptic curve* can be given by an equation

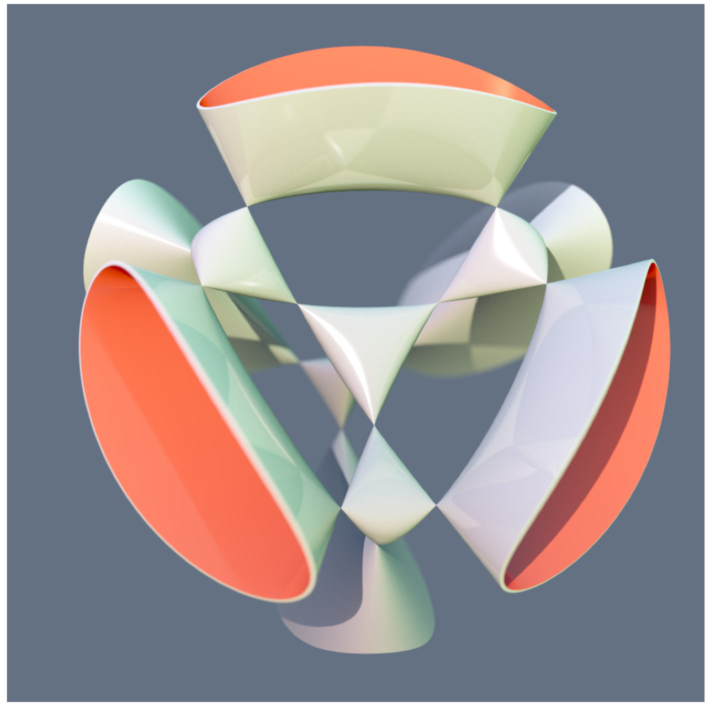
$$E(a,b,c) := (y^2 = x^3 + ax^2 + bx + c) \subset \mathbb{C}^2.$$

The curve $E(a,b,c)$ is smooth iff the cubic has 3 simple roots, that is, when the discriminant $\Delta := 18abc - 4a^3c + a^2b^2 - 4b^3 - 27c^2$ is nonzero. Two such curves are isomorphic iff there is a coordinate change $(x,y) \mapsto (\alpha^2 x + \beta, \alpha^3 y)$ that transforms one equation into the other. All these transformations form a group G and the isomorphism classes of all elliptic curves are in one-to-one correspondence with the orbits of G on $\mathbb{C}^3 \setminus (\Delta = 0)$.

One can describe the orbit space using the j-invariant $j\big(E(a,b,c)\big) := 2^8(a^2 - 3b)^3/\Delta$, defined by Klein (1878). We get a one-to-one correspondence

$$\left\{ \begin{array}{c} \text{set of elliptic curves} \\ \text{modulo isomorphism} \end{array} \right\} \stackrel{j}{\longleftrightarrow} \mathbb{C}.$$

The only sensible compactification of $\mathbb{C}$ is $\mathbb{CP}^1$ and for the point at infinity we should take the nodal curve $y^2 = x^3 + x^2$, which has j-invariant ∞.



A cut Kummer quartic surface. Image generated by Abdelaziz Naït-Merzouk.

Main Steps of a General Moduli Theory

We hope to do something similar with more general algebraic varieties. We proceed in several steps.

Step 1. Identify a class of projective varieties **V** that should have a "good" moduli theory. We aim to prove that such a theory exists for K-positive varieties. For simplicity we fix the dimension and the *canonical volume*

$$\mathrm{vol}(X) := \big(K_X^n\big) = \textstyle\int_X c_1(\omega)^n.$$

It seems that in most other cases there is no "good" compactified moduli theory, unless some additional structure is added on. However, recently it was discovered that many Fano varieties should have a "good" moduli theory; a large group led by C. Xu is working on this.

Step 2. We choose a distinguished class of embeddings $i : X \hookrightarrow \mathbb{P}^N$ for some N. A difficult point is to show that a fixed N works for all $X \in$ **V**. For smooth varieties, this was proved by Matsusaka (1972); the singular case for surfaces was done by Alexeev (1994) and extended to higher dimensions by Hacon–McKernan–Xu (2018).

Focus on the Scientist: Claire Voisin

Claire Voisin, a Clay Senior Scholar in the MSRI program Birational Geometry and Moduli Spaces, has made outstanding contributions to algebraic geometry. Her research is centered on Hodge theory and algebraic cycles, and she has used her insight into these areas to break problems open throughout algebraic geometry.

Claire Voisin

Claire was born in Saint-Leu-la-Forêt, in a family with eight sisters and three brothers. As a child, she loved philosophy, poetry, and painting, as well as mathematics. At 19, she entered the École normale supérieure in Paris. She did her Ph.D. in 1986 with Arnaud Beauville.

Claire spent 30 years as a CNRS researcher at the Institut de Mathématiques de Jussieu in Paris. Since 2016, she has been Professor at the Collège de France. She has been awarded many of the highest honors in mathematics, including the CNRS Gold Medal (2016) and the Shaw Prize (2017). She is a tireless traveler, and she is generous with ideas and suggestions for geometers around the world.

The central theme of Claire's work is Hodge theory, which can be described as the study of complex algebraic geometry by analytic methods. One of the joys of algebraic geometry is that spaces defined by polynomial equations can be studied from multiple points of view (analysis, topology, number theory), with each approach bringing new insights.

Among Claire's best-known results is her solution of the Kodaira problem, showing that not every compact Kähler manifold can be deformed to a smooth projective variety. She proved the generic Green's conjecture on the equations defining algebraic curves.

Claire's current interests include the rationality problem for algebraic varieties. The technique she introduced in 2013, known as "decomposition of the diagonal," revolutionized the subject and led to many further advances. She is also fascinated with hyperkähler manifolds, which have a rich theory at the border of algebraic and analytic geometry.

Beyond mathematics, the center of Claire's life is her five children, and she is rarely away from them for long.

— Burt Totaro

Step 3. In order to get rid of the extra datum of the embedding, we have to take the quotient by $\mathrm{PGL}(N+1,\mathbb{C})$. Following the works of Mumford (1965) and Seshadri (1963–72), the general case was settled by Kollár (1997) and, in the optimal form, by Keel–Mori (1997).

Step 4. In almost all cases, the resulting spaces are not compact, and compactifying them in a "good" way is difficult. The key is to identify the limits of one-parameter families of varieties in $\mathbf{V}$. In our elliptic example we have a nodal curve at infinity. An important discovery of Deligne–Mumford (1969) is that in general we should allow curves with nodes on limits of smooth curves. Their higher dimensional versions of nodes are the *semi-log-canonical* singularities. Very roughly, they are characterized by the volume of the smooth part of X having at most logarithmic growth near the singular points of X.

Step 5. We now have to go back and redo steps 1–3 for (possiby reducible) varieties that have semi-log-canonical singularities and positive K_X. The end result is a compactified moduli space $\overline{\mathcal{M}}_{\mathbf{V}}$.

Step 6. Finally, if everything works out, then we would like to study the properties of $\mathcal{M}_{\mathbf{V}}$, $\overline{\mathcal{M}}_{\mathbf{V}}$ and to use these to prove further theorems. This is another goal of the program and of future research.

Forthcoming Workshops

May 6–10, 2019: *Recent Progress in Moduli Theory*

Aug 15–16, 2019: *Connections for Women: Holomorphic Differentials in Mathematics and Physics*

Aug 19–23, 2019: *Introductory Workshop: Holomorphic Differentials in Mathematics and Physics*

Aug 29–30, 2019: *Connections for Women: Microlocal Analysis*

Sep 3–6, 2019: *Introductory Workshop: Microlocal Analysis*

Oct 14–18, 2019: *Recent Developments in Microlocal Analysis*

Nov 18–22, 2019: *Holomorphic Differentials in Mathematics and Physics*

Summer Graduate Schools

Jun 3–14, 2019: *Commutative Algebra and its Interaction with Algebraic Geometry*

Jun 10–21, 2019: *Random and Arithmetic Structures in Topology*

Jun 24–Jul 5, 2019: *Representation Stability*

Jul 1–13, 2019: *Séminaire de Mathématiques Supérieures 2019: Current Trends in Symplectic Topology*

Jul 1–12, 2019: *Geometric Group Theory*

Jul 8–19, 2019: *Polynomial Method*

Jul 22–Aug 2, 2019: *Recent Topics on Well-Posedness and Stability of Incompressible Fluid and Related Topics*

Jul 29–Aug 9, 2019: *Toric Varieties*

Jul 29–Aug 9, 2019: *Mathematics of Machine Learning*

Jul 29–Aug 9, 2019: h-*Principle*

To find more information about any of these workshops or summer schools, as well as a full list of all upcoming workshops and programs, please visit msri.org/scientific.

Puzzles Column

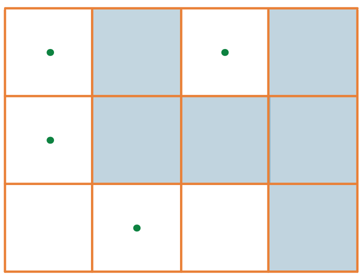
Elwyn Berlekamp, Joe P. Buhler, and Tanya Khovanova

The 21st annual Bay Area Mathematical Olympiad took place on Feb 26, 2019. On March 10, MSRI hosted the awards ceremony, which also featured a special presentation by Mira Bernstein (Chair, Board of Directors, Canada/USA Mathcamp). Problems 3 and 4 below come from BAMO 2019. There are interesting things to say about the origins of the other problems: these will be included (along with solutions!) in the online version of this column at msri.org/emissary.

1. What is the smallest degree of a monic integer polynomial $f(x)$, such that 100 divides $f(n)$ for all integers n?

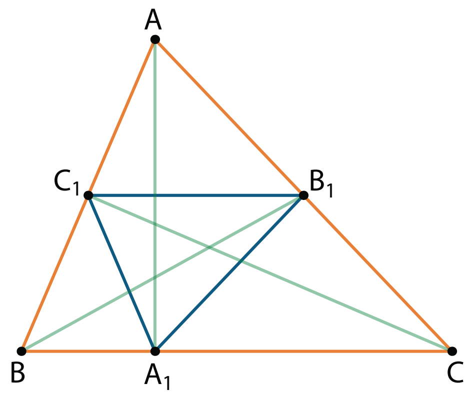
2. Flipping two coins produces three different results: "two heads," "two tails," and "one head and one tail." What is the expected number of these two-coin flips until all three results have been seen?

3. An 8×8 grid of squares starts with all squares being white. You choose any square and color it grey. Then you can repeat the following process for as long as you want: Choose any square that has exactly 1 or 3 grey neighbors, and color that square grey. (Two squares are neighbors if they share an edge.) Is it possible to end up with a grid in which every square is grey?



(To clarify which moves are allowed, in the smaller 3×4 grid above, six squares have been colored grey so far, and the only squares that can be colored now are marked with a dot.)

4. On a triangle ABC there are points A_1, B_1, C_1 on sides opposite the corresponding vertices. The line AA_1 is an altitude, BB_1 is a median, and CC_1 is an angle bisector. Prove that if the triangle $A_1B_1C_1$ is equilateral then so is ABC.



5. Prove that for every positive integer n, there are integers a and b such that

$$3^{3^n} - 1 = a^2 + b^2.$$

6. At time 0, point A is at $(0, 0)$ in the plane, and point B is at $(0, 1)$. Point A starts moving, at speed 1, to the right on the x-axis. Point B starts moving, at speed 1 and always towards A's position. What is the limiting distance between A and B?

2019 Mathical Book Prize Winners Announced

MSRI's Mathical Book Prize recognizes outstanding fiction and literary nonfiction for youth aged 2–18. The prize, now in its fifth year, is selected annually by a committee of pre-K-12 teachers, librarians, mathematicians, early childhood experts, and others. This year's winners are:

Pre-K: *Crash! Boom! A Math Tale* by Robie H. Harris (Candlewick Press)

Grades K–2: *Nothing Stopped Sophie: The Story of Unshakable Mathematician Sophie Germain* by Cheryl Bardoe (Little, Brown Books for Young Readers)

Grades 3–5: *The Miscalculations of Lightning Girl* by Stacy McAnulty (Random House Children's Books).

Grades 6–8: *To the Moon! The True Story of the American Heroes on the Apollo 8 Spaceship* by Jeffrey Kluger and Ruby Shamir (Philomel Books for Young Readers)

The winning titles were announced as part of the Critical Issues in Mathematics Education (CIME) conference at MSRI on March 7. As part of the 2019 National Math Festival, past and present Mathical award-winning authors will speak and host book signings for the public on May 4 in Washington, DC.

The Mathical Book Prize is awarded by MSRI in partnership with the National Council of Teachers of English and the National Council of Teachers of Mathematics, and in coordination with the Children's Book Council. The Mathical list is intended as a resource for educators, parents, librarians, children, and teens. Download the list at mathicalbooks.org.

Download a compete list of current and past winners and honorees at mathicalbooks.org.

2018 Annual Report

We gratefully acknowledge the supporters of MSRI whose generosity allows us to fulfill MSRI's mission to advance and communicate the fundamental knowledge in mathematics and the mathematical sciences; to develop human capital for the growth and use of such knowledge; and to cultivate in the larger society awareness and appreciation of the beauty, power, and importance of mathematical ideas and ways of understanding the world.

This report acknowledges grants and gifts received from January 1–December 31, 2018. In preparation of this report, we have tried to avoid errors and omissions. If any are found, please accept our apologies, and report them to development@msri.org. If your name was not listed as you prefer, let us know so we can correct our records. If your gift was received after December 31, 2018, your name will appear in the 2019 Annual Report. For more information on our giving program, please visit www.msri.org.

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We recognize our most generous and loyal supporters whose leadership and commitment ensure MSRI continues to thrive as one of the world's leading mathematics research centers.

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The Museion Society, named after Musaeum, the Hall of the Muses in ancient Alexandria, recognizes our leadership donors in annual and endowment giving.

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There are many ways to support MSRI with a donation or a planned gift. *A planned gift in the form of a bequest may enable you to make a substantial gift to MSRI, ensuring that the institute remains strong in the future. Planned gifts may help reduce capital gains and estate taxes. Friends who include MSRI in their will or have made other stipulations for a gift from their estate become members of the Gauss Society and receive invitations to our Museion dinners and to a reception during the annual Joint Mathematics Meeting.* ***To learn more about the Gauss Society and other ways to support MSRI****, please contact Annie Averitt, Director of Advancement and External Relations, at 510-643-6056 or aaveritt@msri.org.*

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Individuals can make a significant impact on the life of the Institute, on mathematics, and on scientific research by contributing to MSRI.

Make a Donation to MSRI

Dr. ***Calvin C. Moore*** *is a co-founder of MSRI and the institute's first Deputy Director. He and his wife Doris recently discussed their decision to designate a portion of their estate as a planned gift to MSRI.*

Why did you choose to make a planned gift to MSRI?

Well, specifically to the library. When I was deputy director, one of my major projects was the creation of the library.

You were MSRI's first librarian?

Yes. The challenge then was to get back issues of all the journals that we needed. I went around to people in the mathematics community who were the past editors of the journals and asked them to donate what we needed. The library was built with that kind of donation from other mathematicians. Since then, it's been a pet project of mine. I've been making donations to the library over the years and [the planned gift] is an extension of that.

Why should others make a planned gift?

If you agree with what MSRI is doing, this is a good way to support it. You can contribute more and have a bigger impact. To really ensure the institute's permanence they need to triple or quadruple the endowment. Planned giving is a great way to help do that. It wasn't hard to set up. You need a lawyer to draw up the trust and your last will and testament.

What's MSRI's benefit to the mathematical community?

One of the reasons we were funded initially is because the existing institutes were limited in the number of mathematicians they could accommodate or had specialized focus, and all of them were on the East Coast. Many people felt there needed to be an additional institution located elsewhere in the country that could provide a career shaping experience to more young people than IAS, for example, could accommodate.

MSRI provides young mathematicians the opportunity to interact with the world's leading researchers in a given field. The most accomplished researchers serve as the faculty for semester- or year-long programs and provide guidance and opportunity for collaboration for researchers at the beginning of their careers.

Also, the emphasis on diversity and inclusion is something that was not so much part of the original plan but has become a big part of MSRI's current emphasis. Bringing more minorities and women into mathematics and training them to the highest level is a very good thing. And that's something that appeals to donors, too.

For more information about making a gift, please see the annual donor report on pages 14–15.