**Final Report
on the
Mathematical Sciences Research Institute
2016–17 Activities
supported by
NSA Grant H98230-16-1-0325
Support of Advanced Graduate Students
as Pre-doctoral Members at MSRI
9/06/2016-9/05/2017**

November 2017

Mathematical Sciences Research Institute
NSA Final Report for H98230-16-1-0325

I.   INTRODUCTION

The main scientific activities of MSRI are its Programs and Workshops. MSRI hosts two to four semester-long programs each year. Each program has about forty mathematicians in residence at any given time, including seven to eight graduate students.

Generally, each semester-long program features three workshops. A program begins with a *Connections for Women Workshop*, which is designed to encourage the participation of women in the research activities of the program.   If the area of mathematics is one that traditionally has a large number of women, then the workshop can be used to highlight and to showcase their individual work. However, if the number of women in the field is low, then the workshop is marketed to a wider female audience in an effort to stimulate interest in the area. In addition, another goal is to encourage new connections among the women early in the program as a catalyst for eventual collaborations.   This workshop is then followed by an *Introductory Workshop*, the purpose of which is to introduce the subject to the broader mathematical community. Later during the program, there is a *Topical Workshop*, which is designed to explore some of the themes of the program in greater depth.

In addition to the scientific workshops that run parallel with the programs, MSRI hosts a Hot Topics workshop. These workshops are intended to explore emerging topics in mathematics. (See Section II.B for a brief summary)

MSRI also hosts and co-hosts Education & Outreach Workshops.   These workshops focus on improving the skills of K–12 math teachers. Their descriptions, as well as lists of speakers, talks and participants, can be found on the MSRI web site at http://www.msri.org/web/msri/education/for-k-12-educators.

Another essential activity at MSRI is its series of Summer Graduate Schools which target advanced graduate students in mathematics. During the summer of 2016, MSRI hosted four on-site and two off-site Summer Graduate Schools, with themes ranging from electronic structure theory topics to tropical curves. A complete description can be found at the URL http://www.msri.org/web/msri/scientific/workshops/summer-graduate-school.

Last but not least, each summer since 2007, MSRI has hosted a summer school (MSRI-UP) for undergraduate students with the aim of increasing the number of PhDs among members of under-represented groups. These summer schools are co-funded by the NSA and the NSF. The 2016 MSRI-UP, *Sandpile Groups*, was a successful and popular school, with 18 undergraduate participants. Since MSRI-UP is funded by an independent NSA grant, its report is filed separately. More information about MSRI-UP can be found on our website at http://www.msri.org/web/msri/education/for-undergraduates/msri-up.

II.   OVERVIEW OF ACTIVITIES 2016–17

The year 2016–17 was an exciting one. In Fall 2016, we held a jumbo program: *Geometric Group Theory*, with lead organizer Mark Feighn (Rutgers). In Spring 2017, we held two programs: *Analytic Number Theory*, led by Terence Tao (UCLA), and *Harmonic Analysis*, led by Michael Christ (UC Berkeley) and Michael Lacey (Georgia Tech). MSRI also hosted a small Complementary Program for mathematicians whose interests were not closely related to the three programs. All programs are briefly summarized in Section II. A

There were 250 researchers who participated in these programs for a period of one month or longer. Of those members, there were 34 Postdoctoral Fellows, 50 Research Professors, 120 Research Members, and 46 Program Associates (Graduate Students).

The NSA grant H98230-16-1-0325 funded nine Program Associates (graduate students): five participated in the fall jumbo program, *Geometric Group Theory*, two participated in each of the spring programs, *Analytic Number Theory* and *Harmonic Analysis*.

A. Major Programs and their Associated Workshops

In the list of organizers of each activity, an asterisk (*) denotes lead organizer(s).

Program 1: Geometric Group Theory
August 15, 2016 - December 16, 2016
*Organizers: Ian Agol (University of California, Berkeley), Mladen Bestvina (University of Utah), Cornelia Drutu (University of Oxford), *Mark Feighn (Rutgers University), Michah Sageev (Technion---Israel Institute of Technology), Karen Vogtmann (University of Warwick)*

The field of geometric group theory emerged from Gromov's insight that even mathematical objects such as groups, which are defined completely in algebraic terms, can be profitably viewed as geometric objects and studied with geometric techniques Contemporary geometric group theory has broadened its scope considerably, but retains this basic philosophy of reformulating in geometric terms problems from diverse areas of mathematics and then solving them with a variety of tools. The growing list of areas where this general approach has been successful includes low-dimensional topology, the theory of manifolds, algebraic topology, complex dynamics, combinatorial group theory, algebra, logic, the study of various classical families of groups, Riemannian geometry and representation theory.

The goals of this MSRI program are to bring together people from the various branches of the field in order to consolidate recent progress, chart new directions, and train the next generation of geometric group theorists.

Workshops associated with the *Geometric Group Theory* program:

Workshop 1: Connections for Women: Geometric Group Theory
August 17, 2016 - August 19, 2016
*Organizers: *Ruth Charney (Brandeis University), Indira Chatterji (Université Nice Sophia-Antipolis), Mark Feighn (Rutgers University), Talia Fernós (University of North Carolina)*

This three-day workshop featured talks by six prominent female mathematicians on a wide range of topics in geometric group theory. Each speaker gave two lectures, separated by a break-out session during which participants met in small groups to discuss ideas presented in the first lecture. The workshop was open to all mathematicians.

Workshop 2: Introductory Workshop: Geometric Group Theory
August 22, 2016 - August 26, 2016
*Organizers: Martin Bridson (University of Oxford), Benson Farb (University of Chicago), *Zlil Sela (Hebrew University), Karen Vogtmann (University of Warwick)*

This was an introductory workshop to the MSRI jumbo program Geometric Group Theory held during the Fall Semester of 2016. The purpose of the workshop was to provide an overview of key areas of research to be covered in the program, including an introduction to open problems of current interest.

Workshop 3: Groups acting on CAT(0) spaces
September 27, 2016 - September 30, 2016
*Organizers: Ian Agol (University of California, Berkeley), Pierre-Emmanuel Caprace (Université Catholique de Louvain), Koji Fujiwara (Kyoto University), Alessandra Iozzi (ETH Zürich), * Michah Sageev (Technion---Israel Institute of Technology)*

The theme of the workshop was algebraic, geometric and analytical aspects of groups that act by isometries on spaces of non-positive curvature known as CAT(0) spaces. The world of CAT(0) spaces includes classical spaces such as symmetric spaces and buildings, as well as more avant-garde arrivals, such as CAT(0) cube complex. The workshop brought together researchers studying various aspects of such groups and spaces to discuss recent developments and chart new directions in the field.

Workshop 4: Geometry of mapping class groups and Out (Fn)
October 25, 2016 - October 28, 2016
*Organizers: Yael Algom-Kfir (University of Haifa), *Mladen Bestvina (University of Utah), Richard Canary (University of Michigan), Gilbert Levitt (Université de Caen)*

A four-day workshop with research-level talks on the latest advances in the geometry of mapping class groups and Out(F_n), and spaces on which they act.

Workshop 5: Amenability, coarse embeddability and fixed point properties
December 06, 2016 - December 09, 2016

*Organizers: Goulnara Arzhantseva (University of Vienna), *Cornelia Drutu (University of Oxford), Graham Niblo (University of Southampton), Piotr Nowak (Polish Academy of Sciences)*

The main theme of the workshop is the spectrum of analytic properties running from Kazhdan's property (T) at one end to von Neumann's amenability at the other, that forms a foundational organizing structure for infinite groups and spaces. These properties can be described both analytically, via unitary representation theory, and geometrically, using embedding properties for discrete spaces. Connections with probability and combinatorics was addressed during the meeting.

Program 2: Analytic Number Theory
January 17, 2017 - May 26, 2017
*Organizers: Chantal David (Concordia University), Andrew Granville (Université de Montréal), Emmanuel Kowalski (ETH Zuerich), Philippe Michel (École Polytechnique Fédérale de Lausanne (EPFL)), Kannan Soundararajan (Stanford University), *Terence Tao (University of California, Los Angeles*

Analytic number theory, and its applications and interactions, are currently experiencing intensive progress, in sometimes unexpected directions. In recent years, many important classical questions have seen spectacular advances based on new techniques; conversely, methods developed in analytic number theory have led to the solution of striking problems in other fields.

Recent advances in analytic number theory have had repercussions in various mathematical subjects, such as harmonic analysis (including the Langlands programme), ergodic theory and dynamics (especially on homogenous spaces), additive and multiplicative combinatorics and theoretical computer science (in particular, through the theory of expander graphs).

The MSRI semester program in Spring 2017 focused on the topic of Analytic Number Theory, with workshops and other activities focused on the most impressive recent achievements in this field. We wish not only to give the leading researchers in the area further opportunities to work together, but more importantly to give young people the occasion to learn about these topics, and to give them the tools to achieve the next breakthroughs.

Workshops associated with the *Analytic Number Theory* program:

Workshop 1: Connections for Women: Analytic Number Theory
February 02, 2017 - February 03, 2017
Chantal David (Concordia University), Kaisa Matomäki (University of Turku), Lillian Pierce (Duke University), Kannan Soundararajan (Stanford University), Terence Tao (University of California, Los Angeles)

This workshop consisted of lectures on the current state of research in analytic number theory, given by prominent women and men in the field. The workshop was opened to all graduate students, post-docs, and researchers in areas related to the program; it also included a panel discussion session among female researchers on career issues, as well as other social events.

Workshop 2: Introductory Workshop: Analytic Number Theory
February 06, 2017 - February 10, 2017
*Andrew Granville (Université de Montréal), *Emmanuel Kowalski (ETH Zuerich), Kaisa Matomäki (University of Turku), Philippe Michel (École Polytechnique Fédérale de Lausanne (EPFL))*

There were three minicourses, each three hours long: (1) K. Matomäki and M. Radziwill on multiplicative functions (2) Ph. Michel on trace functions and their applications (3) I. Petrow on Kuznetsov formula, Kloostermania and applications All other talks were one-hour long and targeted to a wide audience. In particular, speakers attempted to give lectures that are also accessible to the participants of the Harmonic Analysis Program.

Workshop 3: Recent developments in Analytic Number Theory
May 01, 2017 - May 05, 2017
*Tim Browning (University of Bristol), Chantal David (Concordia University), Kannan Soundararajan (Stanford University), *Terence Tao (University of California, Los Angeles)*

This workshop focused on presenting the latest developments in analytic number theory, including (but not restricted to) recent advances in sieve theory, multiplicative number theory, exponential sums, arithmetic statistics, estimates on automorphic forms, and the Hardy-Littlewood circle method.

Program 3: Harmonic Analysis
January 17, 2017 to May 26, 2017
*Michael Christ (University of California, Berkeley), Allan Greenleaf (University of Rochester), Steven Hofmann (University of Missouri), *Michael Lacey (Georgia Institute of Technology), Svitlana Mayboroda (University of Minnesota, Twin Cities), Betsy Stovall (University of Wisconsin-Madison), Brian Street (University of Wisconsin-Madison)*

The field of Harmonic Analysis dates back to the 19th century, and has its roots in the study of the decomposition of functions using Fourier series and the Fourier transform. In recent decades, the subject has undergone a rapid diversification and expansion, though the decomposition of functions and operators into simpler parts remains a central tool and theme.

This program brought together researchers representing the breadth of modern Harmonic Analysis and seek to capitalize on and continue recent progress in four major directions:

-Restriction, Kakeya, and Geometric Incidence Problems

-Analysis on Nonhomogeneous Spaces

-Weighted Norm Inequalities

-Quantitative Rectifiability and Elliptic PDE

Many of these areas draw techniques from or have applications to other fields of mathematics, such as analytic number theory, partial differential equations, combinatorics, and geometric measure theory.

Workshop 1: Connections for Women: Harmonic Analysis
January 19, 2017 - January 20, 2017
*Svitlana Mayboroda (University of Minnesota, Twin Cities), *Betsy Stovall (University of Wisconsin-Madison)*

This workshop highlighted the work of several prominent women working in harmonic analysis, including some of the field's rising stars. There was a panel discussion and a contributed poster session. This workshop was open to all, and poster contributions were welcomed from all mathematicians.

Workshop 2: Introductory Workshop: Harmonic Analysis
January 23, 2017 - January 27, 2017
*Allan Greenleaf (University of Rochester), *Michael Lacey (Georgia Institute of Technology), Svitlana Mayboroda (University of Minnesota, Twin Cities), Betsy Stovall (University of Wisconsin-Madison), Brian Street (University of Wisconsin-Madison)*

This week-long workshop served as an introduction for graduate students, postdocs, and other researchers to the main themes of the program. It featured accessible talks by a number of leading harmonic analysts, including several short courses on the core ideas and techniques in the field. There was a problem session, to which all participants were encouraged to contribute.

Workshop 3: Recent Developments in Harmonic Analysis
May 15, 2017 - May 19, 2017
*Michael Christ (University of California, Berkeley), Steven Hofmann (University of Missouri), *Michael Lacey (Georgia Institute of Technology), Betsy Stovall (University of Wisconsin-Madison), Brian Street (University of Wisconsin-Madison)*

Topics for this workshop was drawn from the main research directions of this conference, including:
(1) Restriction, Kakeya, and geometric incidence problems
(2) Analysis on nonhomogenous spaces
(3) Weighted estimates
(4) Quantitative rectifiability and other topics in PDE

Program 4: Complementary Program (2016-17)
The Complementary Program had a limited number of memberships that were open to mathematicians whose interests were not closely related to the core programs; special consideration was given to mathematicians who were partners of an invited member of a core program.

B. Hot Topics Workshop

Hot Topics: Galois Theory of Periods and Applications
March 27, 2017 - March 31, 2017
*Organizers: *Francis Brown (University of Oxford), Clément Dupont (Université de Montpellier), Richard Hain (Duke University), Vadim Vologodsky (University of Oregon)*

Periods are integrals of algebraic differential forms over algebraically-defined domains and are ubiquitous in mathematics and physics. A deep idea, originating with Grothendieck, is that there should be a Galois theory of periods. This general principle provides a unifying approach to several problems in the theory of motives, quantum groups and geometric group theory. This conference brought together leading experts around this subject and cover topics such as the theory of multiple zeta values, modular forms, and motivic fundamental groups.

III.   PARTICIPATION SUMMARY

A. All MSRI Members

The table below indicates the number of participants for the major programs and workshops that took place at MSRI during the 2016–17 academic year.

Time	Activity Type	Activity Title	No. of participants
Fall 2016	Scientific Program	Geometric Group Theory	107
August 17, 2016 - August 19, 2016	Programmatic Workshop	Connections for Women: Geometric Group Theory	92
August 22, 2016 - August 26, 2016	Programmatic Workshop	Introductory Workshop: Geometric Group Theory	148
September 27, 2016 – September 30, 2016	Programmatic Workshop	Groups acting on CAT(0) spaces	140
October 25, 2016 – October 28, 2016	Programmatic Workshop	Geometry of mapping class groups and Out(Fn)	145
December 06, 2016 – December 09, 2016	Programmatic Workshop	Amenability, coarse embeddability and fixed point properties	95
Spring 2017	Scientific Program	Analytic Number Theory	57
February 02, 2017 – February 03, 2017	Programmatic Workshop	Connections for Women: Analytic Number Theory	70
February 06, 2017 – February 10, 2017	Programmatic Workshop	Introductory Workshop: Analytic Number Theory	138

May 01, 2017 – May 05, 2017	Programmatic Workshop	Recent developments in Analytic Number Theory	138
Spring 2017	Scientific Program	Harmonic Analysis	73
January 19, 2017 - January 20, 2017	Programmatic Workshop	Connections for Women: Harmonic Analysis	65
January 23, 2017 – January 27, 2017	Programmatic Workshop	Introductory Workshop: Harmonic Analysis	124
May 15, 2017 – May 19, 2017	Programmatic Workshop	Recent Developments in Harmonic Analysis	106
Academic Year 2016-17	Scientific Program	Complementary Program 2016-17	13
June 11, 2016 – July 24, 2016	MSRI-UP 2016	Sandpile Groups	18
May 30, 2016 - June 11, 2016	Summer Graduate School (2016)	Séminaire de Mathématiques Supérieures 2016: Dynamics of Biological Systems	25
June 13, 2016 - July 24, 2016	Summer Graduate School (2016)	Harmonic Analysis and Elliptic Equations on real Euclidean Spaces and on Rough Sets	51
June 20, 2016 – July 01, 2016	Summer Graduate School (2016)	Mixed Integer Nonlinear Programming: Theory, algorithms and applications	11
July 11, 2016 - July 22, 2016	Summer Graduate School (2016)	An Introduction to Character Theory and the McKay Conjecture	41
July 18, 2016 - July 29, 2016	Summer Graduate School (2016)	Electronic Structure Theory	39
July 25, 2016 - August 05, 2016	Summer Graduate School (2016)	Chip Firing and Tropical Curves	50
March 27, 2017 – March 31, 2017	Other Scientific Workshop	Hot Topics: Galois Theory of Periods and Applications	75

B. NSA supported Program Associates

There were 46 Program Associates (Graduate Students) who participated in the 2016–17 programs. The NSA grant, H98230-16-1-0325, funded nine out of the 46 graduate students, five graduate students in the fall and four in the spring semester. The funded graduate students had their living expenses reimbursed at the rate of $2,000 per month, prorated for the time that they were in residence at MSRI. Since the provisional Indirect Cost rate decreased from the proposed 24.38% to 20.17%, we were able to fund a little bit more person-months of stipends to the Program Associates. Detailed financial information can be found in Section IV.

Below is more information on the nine graduate students supported by this grant:

GEOMETRIC GROUP THEORY

**Abbott,
Carolyn**

Name: Carolyn Abbott
Year of Ph.D: 2017
Institution of Ph.D.: University of Wisconsin - Madison
Dissertation title: Acylindrical actions on hyperbolic spaces
Ph.D. advisor: Tullia Dymarz

Mentor while at MSRI: Tullia Dymarz

Program Associate's comments:

I worked on many projects at MSRI:

- (joint with F. Dahmani) We proved that acylindrically hyperbolic groups have property $P_{\{naive\}}$, and gave applications of this to the group's C^*-algebra as well as to groups rings of the group.
- (joint with D. Hume) We proved that there are infinitely many quasi-isometry classes of groups that do not admit largest acylindrical actions on hyperbolic spaces, using small cancellation theory. We also gave a bound on the contraction constants of generalized loxodromic elements of a small cancellation group.
- (joint with J. Behrstock and M. Durham) We proved that all hierarchically hyperbolic groups admit a largest acylindrical action on a hyperbolic space. We also classified all stable subgroups of a hierarchically hyperbolic group.

My experience at MSRI was invaluable. In addition to meeting many new people with similar research interests, I was able to start new research projects and make significant progress on old projects. I organized a seminar (the Out(F_n)—complexes seminar), which allowed to me learn about an area that I had been interested in for a while. That seminar has led to collaborations and projects relating to Out(F_n)—complexes which would not have been possible without being at MSRI. I was also on the job market in the fall, and those of us applying for jobs organized an informal weekly meeting where we would meet for a couple hours, work on our applications, ask questions, read each other's research/teaching statements, etc. That was incredibly helpful, and really helped foster a sense of community.

Cahill, Nicholas

Name: Nicholas Cahill
Year of Ph.D: 2018
Institution of Ph.D.: University of Utah
Ph.D. advisor: Mladen Bestvina

Mentor while at MSRI: Mladen Bestvina

Program Associate's comments:

At MSRI I was working in collaboration on studying contracting properties of geodesic subspaces in Outer Space, and on group-theoretic properties of Random Groups in the Gromov model esp. in comparison with latices in rank 1 lie groups. My experience at MSRI was massively beneficial, having the ability to network broadly and openly with so many different highly knowledgable people made it easy to cut through to the core of my questions and not spend so much time digging through literature. I think that such an environment is tremendously valuable to mathematics.

Farre, James

Name: James Farre
Year of Ph.D.: 2018
Institution of Ph.D.: University of Utah
Ph.D. advisor: Kenneth Bromberg

Mentor while at MSRI: Kenneth Bromberg

Program Associate's comments:

While at MSRI, I studied bounded cohomology of discrete groups, hyperbolic three-manifolds, and geometry of groups. I established connections and started to work collaboratively with the geometric group theory community. It was extremely beneficial for me to meet and engage with leaders in the field, have a very pleasant place to work with excellent programming, and have very few distractions; it was an immersive and exciting time for me.

Murray, Devin

Name: Devin Murray
Year of Ph.D.: 2018
Institution of Ph.D.: Brandeis
Dissertation title (if available): The morse boundary of CAT(0) groups
Ph.D. advisor: Ruth Charney

Mentor while at MSRI: Ruth Charney

Program Associate's comments:

I worked with Ruth Charney on a new paper which we finished writing shortly after our stay there. The collaboration and proximity of so many other mathematicians working in the same field was extremely beneficial for developing a program of research that I'm now carrying out.

Wigglesworth, Derrick

Name: Derrick Wigglesworth
Year of Ph.D.: 2018
Institution of Ph.D.: University of Utah
Ph.D. advisor: Mladen Bestvina

Mentor while at MSRI: Mladen Bestvina

Program Associate's comments:

While at MSRI, I started several new collaborations and completed one ongoing project. The project I completed while at MSRI was a study of distortion for abelian subgroups of the outer automorphism group of a free group, Out(Fn). I began a collaboration with Radhika Gupta with the goal of better understanding the action of Out(Fn) on the cyclic splitting complex. We have a forthcoming paper with some results about this action, and the work we have done thus far has led us to several new problems that we are currently working on.

My time at MSRI was extremely beneficial for several reasons. First, I had the opportunity to meet and get acquainted with experts in my field from all over the world. This has already proved helpful as I am currently in the midst of the job application process. As I already mentioned, my interactions with members at MSRI led to collaborations and an exchange of ideas that have been very helpful in driving my research, and expanding it into other areas.

ANALYTIC NUMBER THEORY

Lowry-Duda, David

Name: David Lowry-Duda
Year of Ph.D.: 2017
Institution of Ph.D.: University of Utah
Dissertation title: On some variants of the Gauss circle problem
Ph.D. advisor: Jeffrey Hoffstein

Mentor while at MSRI: Jeffrey Hoffstein

Program Associate's comments:

I began a variety of projects while at MSRI. I examined the distribution of primes along quadratics while learning about the circle method with some of the masters of the circle method. This project ultimately failed to improve existing results, but it was very useful. I studied the number of points contained within hyperbolic lattices (a major part of the dissertation), normal lattices (culminating in two recently submitted papers), and irregular lattices (culminating in a paper with new collaborators, which we hope to submit soon). Remarkably, though the descriptions are similar, the techniques and collaborators are different for each of these projects. I value my experiences at MSRI, and I would absolutely return if given the opportunity. I created so many connections and learned so much while there.

Walker, Alexander

Name: Alexander Walker
Year of Ph.D.: 2017
Institution of Ph.D.: Brown University
Dissertation title: Sums of Coefficients of Modular Forms and the Gauss Circle Problem
Ph.D. advisor: Jeffrey Hoffstein

Mentor while at MSRI: Jeffrey Hoffstein

Program Associate's comments:

While at MSRI I prepared three papers. Two of these papers concern the generalized Gauss circle problem and have been submitted to journals. The third, joint with new collaborator Aled Walker, is essentially written and will hopefully be submitted soon.

My time at MSRI was immensely fruitful. Not only has the research I conducted there improved my thesis, it has led to connections across the world which will help me in future professional endeavors. I have learned a lot from other researchers at MSRI and hope to take advantage of what I have learned in future projects.

HARMONIC ANALYSIS

Fraser, Robert

Name: Robert Fraser
Year of Ph.D.: 2018
Institution of Ph.D.: University of British Columbia
Ph.D. advisor: Malabika Pramanik

Mentor while at MSRI: Malabika Pramanik

Program Associate's comments:

While I was at MSRI, I made revisions to an old paper that I had co-authored with Malabika Pramanik. Malabika and I also worked on a new paper.

As for my work with the other participants, I began a new project with Shaoming Guo and Malabika Pramanik. We are looking at subsets of the real numbers that contain 3 points of the form x, x+t, x+t^2.

I gained a lot from the MSRI program. It was invaluable to meet graduate students and postdocs at other institutions, and I learned a lot from all of the talks I attended. Probably the most valuable thing I gained was a sense for the popular research directions in harmonic analysis, e.g. sparse bounds for operators.
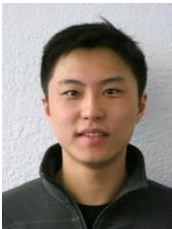
Li, Weilin

Name: Weilin Li
Year of Ph.D.: 2018
Institution of Ph.D.: University of Maryland
Ph.D. advisor: John Benedetto and Wojciech Czaja

Mentor while at MSRI: Kasso Okoudjou

Program Associate's comments:

The facilities and support at MSRI were amazing and I thoroughly enjoyed the experience. While at MSRI, I worked on two separate projects. The first is related to super-resolution problem from imaging sciences, where the aim was to determine the best possible approximation rate and best possible algorithm for recovering sparse, but possibly densely distributed, discrete measures from a small number of Fourier coefficients. The second project is related to the construction of directional time-frequency representations that are well-localized in the phase space and extract directional information from data sets, and their applications to machine learning.