SERGEY YUZVINSKY (p.1) 02-04-03

$S = \mathbb{C}[x_1,\ldots,x_\ell]$ $\quad P_1,\ldots,P_\ell \in S_d$

$P_1(t)$ continuous at t, $t \in [0,1]$, $I_t = (P_1,\ldots,P_\ell)$, $R_t = S/I_t$, Kdim S/I_t = const.

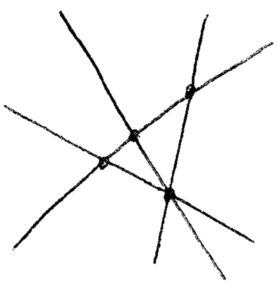
Question: Can $\mathrm{depth}_{S_t} R_t$ go down when $t \to 0$?

Let $Q \in S_n$; moreover $Q = \prod \alpha_i$, $\alpha_i \in S_1$, $\alpha_i \not\sim \alpha_j$, $i \neq j$.

$I = \left(\frac{\partial Q}{\partial x_i} \mid i = 1,\ldots,\ell\right)$, $Q = 0$ is the defining eq^n of $\bigcup_{i=1}^{n} H_i$, $H_i = \mathrm{Ker}\,\alpha_i$

Kdim $S/I = \ell - 2$

$\ell = 3$



General pos: depth $S/I = 0$ $\quad \boxed{\cap H_i = 0}$

$A = \{H_i\}$ = arrangement

The most "special": depth $= \ell - 2$

S/I is CM

$$0 \to \underbrace{\{\theta \in \mathrm{Der}\,S \mid \theta Q = 0\}}_{D_0} \to S^\ell \xrightarrow{\cdot Q} S \to S/I \to 0$$

$$S^\ell \cong \mathrm{Der}_{\mathbb{C}} S = \left\langle \tfrac{\partial}{\partial x_i} \right\rangle_{i=1}^{\ell}$$

$D(A) = D_0 \oplus S\theta_E$ where $\theta_E = \sum_{i=1}^{\ell} x_i \,\partial/\partial x_i$

$D(A) = \{\theta \in \mathrm{Der}_{\mathbb{C}} S \mid \theta Q \in QS\}$ = all poly. vector fields on $\mathbb{C}^\ell$ s.t. at pts of any H_i the vectors $\parallel H_i$.

CM case $\Longleftrightarrow$ D_0 = free $\Longleftrightarrow$ $D(A)$ is free

Combinatorics of A = matroid

Terao's Conjecture: the freeness is defined by combinatorics

Known: for fixed comb. type free arr's form Zariski open set.

(SY p2)

$\boxed{\mathrm{II}}$ $M = \mathbb{C}^{\ell} \setminus \bigcup_{i=1}^{n} H_i$ $\qquad M_{0,n}$ $\qquad H^*(M;\mathbb{C})$

<u>Example</u> $x-y,\ x-z,\ y-z$

Create a ring using A. $E = \Lambda(\underset{H_1}{e_1},\dots,\underset{H_n}{e_n})$, $\partial : E \to E$, $\partial e_i = 1$
$\mathbb{C}$ linear, signed, Leibniz
$I = \langle \partial e_{i_1 \dots i_p} \mid \{H_{i_1},\dots,H_{i_p}\} \text{ dependent} \rangle$ $\qquad \boxed{A = E/I}$

Orlik-Solomon (OS) algebra
<u>Theorem</u> Arnold-Brieskorn-O-S
$A \cong H^*(M;\mathbb{C})$ via $e_i \longmapsto \left[\frac{d\alpha_i}{\alpha_i}\right] \in H^*(M,\mathbb{C})$

Consider A as a cochain complex via $d_a : A_p \to A_{p+1}$, $a \in A_1 = E_1$, $d_a(x) = xa$. $\quad H^*(\underline{A,a})$?

(Ar.-Av-He) $\quad \{a \in E_1 \mid H^*(A,a) \neq 0\} = \mathrm{Sing}\, A$
<u>Sing A is written</u> in E-P-Y (linear space)

$\mathrm{Sing}\, A$ is filtered by $\mathrm{Sing}_p(A) = \{a \in E_1 \mid H^p(A,a) \neq 0\}$; varieties

<u>Problem</u>: study Sing_p by means of <u>comm. algebra</u>
Lot is known for $p = 1$ $\qquad p > 1$ 2 results:
① [Libgober, Cohen-Orlik] Irr. components are linear
② $\mathrm{Sing}_p \subset \mathrm{Sing}_{p+1}$ [EPY]

In fact, $\underset{a \in E_1}{H^p(A,a)} \simeq \mathrm{Tor}_{\ell-p}^{\overbrace{\mathbb{C}[x_1,\dots,x_n]}^{\bar{S}}}(\mathcal{F}_{m_a}, \overset{\bar{S}/m_a}{\overset{\approx}{\mathbb{C}}})$

$\mathcal{F}$ is an $\bar{S}$-module which is BGG-dual to the linear injective resol. of A as an E-mod.

(SY p.3)

Resol. of $\mathcal{F}$ is $0 \to A_0 \otimes \bar{S} \to \cdots \to A_p \otimes_{\mathbb{C}} \bar{S} \to A_{p+1} \otimes \bar{S} \to \cdots \to A_\ell \otimes \bar{S} \to \mathcal{F} \to 0$