Restricted Uniqueness and Stability For a Formally Determined Hyperbolic Inverse Problem with a point source

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For any $q \in C^4(\mathbb{R}^3)$, consider the initial value problem

$$u_{tt} - \Delta u + q(x)u = \delta(x,t), \qquad (x,t) \in \mathbb{R}^3 \times \mathbb{R}$$
$$u(x,t) = 0 \text{ for } t < 0.$$

Choose $0 < R_i < R_o < T$, define the inner and outer cylinders

$$S_i = \{(x,t) \,:\, |x| = R_i,\ R_i \leq t \leq T\}, \qquad S_o = \{(x,t) \,:\, |x| = R_o,\ R_o \leq t \leq T\},$$

and the forward map (data map)

$$F \,:\, C^4(\mathbb{R}^3) \to C^1(S_i) \times C^1(S_i) \times C^1(S_o) \times C^1(S_o)$$
$$F(q) = (u|_{S_i}, u_r|_{S_i}, u|_{S_o}, u_r|_{S_o})\,.$$

We prove that if $F(q_1) = F(q_2)$ then $q_1 = q_2$ on the region $B = \{x \,:\, R_i \leq |x| \leq R_o\}$ provided

- $R_i > T/2$ (e.g. $R_i = 0.6T$);
- $R_o < T + \frac{R_i}{4} - \frac{T^2}{4R_i}$ (e.g. $R_o = 0.73T$);
- $q_1, q_2 \in C^4(\mathbb{R}^3)$;
- $\|\partial_\theta q_j\|_{L^2(B)} \leq K(R_i, R_o, T)\|q_j\|_{L^2(B)}$ for $j = 1, 2$, where ∂_θ represents the angular derivatives (unit length vector fields orthogonal to the radial vector field) and $K(R_i, R_o, T)$ is a predetermined positive constant.

We do not assume any knowledge of the q_j outside $R_i \leq |x| \leq R_o$, we make no smallness assumptions on the q_j or the region. We do impose a restriction on the variation in q_j in the angular directions. If we do assume that $q_1 = q_2$ outside B then we do not need to know u_r on S_i and S_o to prove the uniqueness. We also have a stability result and similar results for a spherical source.