Quantitative Stratification and regularity for harmonic maps and minimal currents

Aaron Naber
Massachusetts Institute of Technology – MIT

In this talk I'll discuss one of two new papers joint with Jeff Cheeger. The papers are devoted to new techniques for taking ineffective local behavior, e.g. tangent cones or tangent maps, and deriving from this effective estimates on regularity. The primary applications I will discuss are those for harmonic maps between Riemannian manifolds, and for minimal currents. For minimizing harmonic maps f:M->N a consequence of the results are L^p estimates for the gradient of f, with p>2, and L^q estimates for the Hessian of f, with q>1. The estimates are sharp, and are the first estimates on the gradient to break the L2 barrier and the first estimates to show that the hessian of f is even measurable. In fact, the estimates are much stronger and give L^p bounds for the regularity scale r_f(x)\equiv max{r>0:sup_{B_r(x)}r|\nabla f|+r2|\nabla2 f|\leq 1}, which controls the size of neighborhoods for which f is smoothly bounded. For minimizing hypersurfaces we analogously prove L^p estimates for p<7 for the second fundamental form and its regularity scale. The proofs include a new quantitative dimension reduction, that in the process stengthens hausdorff estimates on singular sets to minkowski estimates.